You can model and analyze real-world situations by using algebra. In this unit, you will solve and graph linear equations and inequalities and use matrices.

Chapter 1
Solving Equations and Inequalities

Chapter 2
Linear Relations and Functions

Chapter 3
Systems of Equations and Inequalities

Chapter 4
Matrices
“‘Buying a home,’ says Housing and Urban Development Secretary Andrew Cuomo, ‘is the most expensive, most complicated and most intimidating financial transaction most Americans ever make.’” In this project, you will be exploring how functions and equations relate to buying a home and your income.

Log on to www.algebra2.com/webquest. Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 1.
Solving Equations and Inequalities

What You’ll Learn

- **Lesson 1-1** Simplify and evaluate algebraic expressions.
- **Lesson 1-2** Classify and use the properties of real numbers.
- **Lesson 1-3** Solve equations.
- **Lesson 1-4** Solve absolute value equations.
- **Lessons 1-5 and 1-6** Solve and graph inequalities.

Why It’s Important

Algebra allows you to write expressions, equations, and inequalities that hold true for most or all values of variables. Because of this, algebra is an important tool for describing relationships among quantities in the real world. For example, the angle at which you view fireworks and the time it takes you to hear the sound are related to the width of the fireworks burst. A change in one of the quantities will cause one or both of the other quantities to change.

In Lesson 1-1, you will use the formula that relates these quantities.

Key Vocabulary

- order of operations (p. 6)
- algebraic expression (p. 7)
- Distributive Property (p. 12)
- equation (p. 20)
- absolute value (p. 28)
## Prerequisite Skills
To be successful in this chapter, you’ll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 1.

### For Lessons 1-1 through 1-3

<table>
<thead>
<tr>
<th>Problem</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $20 - 0.16$</td>
<td>$19.84$</td>
</tr>
<tr>
<td>2. $-3.01 - 14.5$</td>
<td>$-17.51$</td>
</tr>
<tr>
<td>3. $\frac{1}{4} - \frac{2}{3}$</td>
<td>$-\frac{5}{12}$</td>
</tr>
<tr>
<td>4. $-7\frac{1}{2} + 5\frac{1}{3}$</td>
<td>$-2\frac{1}{3}$</td>
</tr>
<tr>
<td>5. $\frac{3}{5} + (-6)$</td>
<td>$-5.6$</td>
</tr>
<tr>
<td>6. $-11\frac{5}{8} - \left(-4\frac{3}{7}\right)$</td>
<td>$-5\frac{5}{56}$</td>
</tr>
<tr>
<td>7. $(0.15)(3.2)$</td>
<td>$0.48$</td>
</tr>
<tr>
<td>8. $(1.2) \div (-1.1)$</td>
<td>$-1.09$</td>
</tr>
<tr>
<td>9. $-4 \div \frac{3}{2}$</td>
<td>$-\frac{8}{3}$</td>
</tr>
<tr>
<td>10. $\left(-\frac{3}{4}\right) \left(-3\frac{1}{5}\right)$</td>
<td>$2.55$</td>
</tr>
</tbody>
</table>

### For Lesson 1-1

<table>
<thead>
<tr>
<th>Problem</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>17. $2^3$</td>
<td>$8$</td>
</tr>
<tr>
<td>18. $5^3$</td>
<td>$125$</td>
</tr>
<tr>
<td>19. $(-7)^2$</td>
<td>$49$</td>
</tr>
<tr>
<td>20. $(-1)^3$</td>
<td>$-1$</td>
</tr>
<tr>
<td>21. $(-0.8)^2$</td>
<td>$0.64$</td>
</tr>
<tr>
<td>22. $-(1.2)^2$</td>
<td>$-1.44$</td>
</tr>
<tr>
<td>23. $\left(\frac{2}{3}\right)^2$</td>
<td>$\frac{4}{9}$</td>
</tr>
<tr>
<td>24. $\left(-\frac{4}{11}\right)^2$</td>
<td>$\frac{16}{121}$</td>
</tr>
</tbody>
</table>

### For Lesson 1-5

<table>
<thead>
<tr>
<th>Problem</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>25. $-5 &lt; -7$</td>
<td>True</td>
</tr>
<tr>
<td>26. $6 &gt; -8$</td>
<td>True</td>
</tr>
<tr>
<td>27. $-2 \geq -2$</td>
<td>True</td>
</tr>
<tr>
<td>28. $-3 \leq -3.01$</td>
<td>True</td>
</tr>
<tr>
<td>29. $-9.02 &lt; -9.2$</td>
<td>True</td>
</tr>
<tr>
<td>30. $\frac{1}{5} &lt; \frac{1}{8}$</td>
<td>True</td>
</tr>
<tr>
<td>31. $\frac{2}{5} \geq \frac{16}{40}$</td>
<td>True</td>
</tr>
<tr>
<td>32. $\frac{3}{4} &gt; 0.8$</td>
<td>True</td>
</tr>
</tbody>
</table>

---

**Foldables Study Organizer**

Make this Foldable to help you organize information about relations and functions. Begin with one sheet of notebook paper.

**Step 1** Fold

Fold lengthwise to the holes.

**Step 2** Cut and Label

Label the columns as shown.

**Reading and Writing** As you read and study the chapter, write notes, examples, and graphs in each column.
Expressions and Formulas

What You’ll Learn

• Use the order of operations to evaluate expressions.
• Use formulas.

Vocabulary

• order of operations
• variable
• algebraic expression
• formula

How are formulas used by nurses?

Intravenous or IV fluid must be given at a specific rate, neither too fast nor too slow. A nurse setting up an IV must control the flow rate, in drops per minute. They use the formula \( F = \frac{V \times d}{t} \), where \( V \) is the volume of the solution in milliliters, \( d \) is the drop factor in drops per milliliter, and \( t \) is the time in minutes.

Suppose a doctor orders 1500 milliliters of IV saline to be given over 12 hours, or 12 \( \times \) 60 minutes. Using a drop factor of 15 drops per milliliter, the expression \( \frac{1500 \times 15}{12 \times 60} \) gives the correct flow rate for this patient’s IV.

ORDER OF OPERATIONS

A numerical expression such as \( \frac{1500 \times 15}{12 \times 60} \) must have exactly one value. In order to find that value, you must follow the order of operations.

Key Concept

Order of Operations

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>Evaluate expressions inside grouping symbols, such as parentheses, ( ), brackets, [ ], braces, { }, and fraction bars, as in ( \frac{5 + 7}{2} ).</td>
</tr>
<tr>
<td>Step 2</td>
<td>Evaluate all powers.</td>
</tr>
<tr>
<td>Step 3</td>
<td>Do all multiplications and/or divisions from left to right.</td>
</tr>
<tr>
<td>Step 4</td>
<td>Do all additions and/or subtractions from left to right.</td>
</tr>
</tbody>
</table>

Example 1 Simplify an Expression

Find the value of \( [2(10 - 4)^2 + 3] + 5 \).

\[
[2(10 - 4)^2 + 3] + 5 = [2(6)^2 + 3] + 5 \quad \text{First subtract 4 from 10.}
\]

\[
= [2(36) + 3] + 5 \quad \text{Then square 6.}
\]

\[
= (72 + 3) + 5 \quad \text{Multiply 36 by 2.}
\]

\[
= 75 + 5 \quad \text{Add 72 and 3.}
\]

\[
= 15 \quad \text{Finally, divide 75 by 5.}
\]

The value is 15.
Thought and Discuss

1. Simplify $\frac{8}{11002} \div \frac{2}{11003} + \frac{4}{11001}$. Use a graphing calculator.
2. Describe the procedure the calculator used to get the answer.
3. Where should parentheses be inserted in $\frac{8}{11002} \div \frac{2}{11003} + \frac{4}{11001}$ so that the expression has each of the following values?
   a. $-10$
   b. $29$
   c. $-5$
4. Evaluate $18^2 \div (2 \times 3)$ using your calculator. Explain how the answer was calculated.
5. If you remove the parentheses in Exercise 4, would the solution remain the same? Explain.

Variables are symbols, usually letters, used to represent unknown quantities. Expressions that contain at least one variable are called algebraic expressions. You can evaluate an algebraic expression by replacing each variable with a number and then applying the order of operations.

**Example 2** Evaluate an Expression
Evaluate $x^2 - y(x + y)$ if $x = 8$ and $y = 1.5$.

$$x^2 - y(x + y) = 8^2 - 1.5(8 + 1.5)$$
$$= 64 - 1.5(8 + 1.5)$$
$$= 64 - 1.5(9.5)$$
$$= 64 - 14.25$$
$$= 49.75$$

The value is 49.75.

**Example 3** Expression Containing a Fraction Bar
Evaluate $\frac{a^3 + 2bc}{c^2 - 5}$ if $a = 2$, $b = -4$, and $c = -3$.

The fraction bar acts as both an operation symbol, indicating division, and as a grouping symbol. Evaluate the expressions in the numerator and denominator separately before dividing.

$$\frac{a^3 + 2bc}{c^2 - 5} = \frac{2^3 + 2(-4)(-3)}{(-3)^2 - 5}$$
$$= \frac{8 + 24}{9 - 5}$$
$$= \frac{32}{4}$$

The value is 8.
A *formula* is a mathematical sentence that expresses the relationship between certain quantities. If you know the value of every variable in the formula except one, you can find the value of the remaining variable.

**Example 4 Use a Formula**

**GEOMETRY** The formula for the area \( A \) of a trapezoid is \( A = \frac{1}{2}h(b_1 + b_2) \), where \( h \) represents the height, and \( b_1 \) and \( b_2 \) represent the measures of the bases. Find the area of the trapezoid shown below.

Substitute each value given into the formula. Then evaluate the expression using the order of operations.

\[
A = \frac{1}{2}h(b_1 + b_2) \quad \text{Area of a trapezoid}
\]

\[
= \frac{1}{2}(10)(16 + 52) \quad \text{Replace } h \text{ with 10, } b_1 \text{ with 16, and } b_2 \text{ with 52.}
\]

\[
= \frac{1}{2}(10)(68) \quad \text{Add 16 and 52.}
\]

\[
= 5(68) \quad \text{Divide 10 by 2.}
\]

\[
= 340 \quad \text{Multiply 5 by 68.}
\]

The area of the trapezoid is 340 square inches.

**Check for Understanding**

**Concept Check**

1. Describe how you would evaluate the expression \( a + b[(c + d) \div e] \) given values for \( a, b, c, d, \) and \( e \).

2. **OPEN ENDED** Give an example of an expression where subtraction is performed before division and the symbols ( ), [ ], or { } are not used.

3. Determine which expression below represents the amount of change someone would receive from a $50 bill if they purchased 2 children’s tickets at $4.25 each and 3 adult tickets at $7 each at a movie theater. Explain.
   
   a. \( 50 - 2 \times 4.25 + 3 \times 7 \)
   
   b. \( 50 - (2 \times 4.25 + 3 \times 7) \)
   
   c. \( (50 - 2 \times 4.25) + 3 \times 7 \)
   
   d. \( 50 - (2 \times 4.25) + (3 \times 7) \)

**Guided Practice** Find the value of each expression.

4. \( 8(3 + 6) \)

5. \( 10 - 8 \div 2 \)

6. \( 14 \cdot 2 - 5 \)

7. \( [9 + 3(5 - 7)] \div 3 \)

8. \( [6 - (12 - 8)^2] \div 5 \)

9. \( \frac{17(2 + 26)}{4} \)

Evaluate each expression if \( x = 4, y = -2, \) and \( z = 6. \)

10. \( z - x + y \)

11. \( x + (y - 1)^3 \)

12. \( x + [3(y + z) - y] \)
**Application**

**BANKING** For Exercises 13–15, use the following information.

Simple interest is calculated using the formula $I = ptr$, where $p$ represents the principal in dollars, $r$ represents the annual interest rate, and $t$ represents the time in years. Find the simple interest $I$ given each of the following values.

13. $p = $1800, $r = 6\%$, $t = 4$ years
14. $p = $5000, $r = 3.75\%$, $t = 10$ years
15. $p = $31,000, $r = 2\frac{1}{2}\%$, $t = 18$ months

**Practice and Apply**

Find the value of each expression.

16. $18 + 6 \div 3$
17. $7 - 20 \div 5$
18. $3(8 + 3) - 4$
19. $(6 + 7)2 - 1$
20. $2(6^2 - 9)$
21. $-2(3^2 + 8)$
22. $2 + 8(5) \div 2 - 3$
23. $4 + 64 \div (8 \times 4) + 2$
24. $[38 - (8 - 3)] \div 3$
25. $10 - [5 + 9(4)]$
26. $1 - [30 \div (7 + 3(-4))]$
27. $12 + [10 \div (11 - 3(2))]$
28. $\frac{1}{3}(4 - 7^2)$
29. $\frac{1}{2}[9 + 5(-3)]$
30. $\frac{16(9 - 22)}{4}$
31. $\frac{45(4 + 32)}{10}$
32. $0.3(1.5 + 24) \div 0.5$
33. $1.6(0.7 + 3.3) \div 2.5$
34. $\frac{1}{5} - \frac{20(81 + 9)}{25}$
35. $\frac{12(52 + 2^2)}{6} - \frac{2}{3}$

36. **BICYCLING** The amount of pollutants saved by riding a bicycle rather than driving a car is calculated by adding the organic gases, carbon monoxide, and nitrous oxides emitted. To find the pounds of pollutants created by starting a typical car 10 times and driving it for 50 miles, find the value of the expression \((52.84 \times 10) + (5.955 \times 50)\) divided by 454.

37. **NURSING** Determine the IV flow rate for the patient described at the beginning of the lesson by finding the value of \(\frac{1500 \times 15}{12 \times 60}\).

Evaluate each expression if $w = 6$, $x = 0.4$, $y = \frac{1}{2}$, and $z = -3$.

38. $w + x + z$
39. $w + 12 \div z$
40. $w(8 - y)$
41. $z(x + 1)$
42. $w - 3x + y$
43. $5x + 2z$
44. $z^4 - w$
45. $(5 - w)^2 + x$
46. $\frac{5wx}{z}$
47. $\frac{2x - 15x}{3y}$
48. $(x - y)^2 - 2wz$
49. $\frac{1}{y} + \frac{1}{w}$

50. **GEOMETRY** The formula for the area $A$ of a circle with diameter $d$ is $A = \pi \left(\frac{d}{2}\right)^2$. Write an expression to represent the area of the circle.

51. Find the value of $ab^n$ if $n = 3$, $a = 2000$, and $b = -\frac{1}{5}$.

Extra Practice

Homework Help

<table>
<thead>
<tr>
<th>For Exercises</th>
<th>See Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>16–17</td>
<td>1, 3</td>
</tr>
<tr>
<td>38–50</td>
<td>2, 3</td>
</tr>
<tr>
<td>51–54</td>
<td>4</td>
</tr>
</tbody>
</table>

More About... Bicycling

In order to increase awareness and acceptance of bicycling throughout the country, communities, corporations, clubs, and individuals are invited to join in sponsoring bicycling activities during the month of May, National Bike Month.

Source: League of American Bicyclists

Visit www.algebra2.com/self_check_quiz for more practice and tools to help you succeed.
52. **MEDICINE** Suppose a patient must take a blood pressure medication that is dispensed in 125-milligram tablets. The dosage is 15 milligrams per kilogram of body weight and is given every 8 hours. If the patient weighs 25 kilograms, how many tablets would be needed for a 30-day supply? Use the formula 

\[ n = \frac{24d}{[8(b \times 15 \div 125)]} \]

where \( n \) is the number of tablets, \( d \) is the number of days the supply should last, and \( b \) is the body weight of the patient in kilograms.

53. **MONEY** In 1950, the average price of a car was about $2000. This may sound inexpensive, but the average income in 1950 was much less than it is now. To compare dollar amounts over time, use the formula 

\[ V = \frac{A}{S}C \]

where \( A \) is the old dollar amount, \( S \) is the starting year’s Consumer Price Index (CPI), \( C \) is the converting year’s CPI, and \( V \) is the current value of the old dollar amount. Buying a car for $2000 in 1950 was like buying a car for how much money in 2000?

**Online Research** 

54. **FIREWORKS** Suppose you are about a mile from a fireworks display. You count 5 seconds between seeing the light and hearing the sound of the firework display. You estimate the viewing angle is about 4°. Using the information at the left, estimate the width of the firework display.

55. **CRITICAL THINKING** Write expressions having values from one to ten using exactly four 4s. You may use any combination of the operation symbols \(+, −, ×, ÷\), and/or grouping symbols, but no other numbers are allowed. An example of such an expression with a value of zero is \((4 + 4) − (4 + 4)\).

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How are formulas used by nurses?**

Include the following in your answer:
• an explanation of why a formula for the flow rate of an IV is more useful than a table of specific IV flow rates, and
• a description of the impact of using a formula, such as the one for IV flow rate, incorrectly.

57. Find the value of \(1 + 3(5 − 17) ÷ 2 × 6\).

\[ \text{A} \quad -4 \quad \text{B} \quad 109 \]
\[ \text{C} \quad -107 \quad \text{D} \quad -144 \]

58. The following are the dimensions of four rectangles. Which rectangle has the same area as the triangle at the right?

\[ \text{A} \quad 1.6 \text{ ft by 25 ft} \quad \text{B} \quad 5 \text{ ft by 16 ft} \]
\[ \text{C} \quad 3.5 \text{ ft by 4 ft} \quad \text{D} \quad 0.4 \text{ ft by 50 ft} \]

**Standardized Test Practice**

59. \(\sqrt{9}\)

60. \(\sqrt{16}\)

61. \(\sqrt{100}\)

62. \(\sqrt{169}\)

63. \(-\sqrt{4}\)

64. \(-\sqrt{25}\)

65. \(\sqrt{\frac{1}{9}}\)

66. \(\sqrt{\frac{36}{49}}\)
Vocabulary
- real numbers
- rational numbers
- irrational numbers

Manufacturers often offer coupons to get consumers to try their products. Some grocery stores try to attract customers by doubling the value of manufacturers’ coupons. You can use the Distributive Property to calculate these savings.

REAL NUMBERS All of the numbers that you use in everyday life are real numbers. Each real number corresponds to exactly one point on the number line, and every point on the number line represents exactly one real number.

Real numbers can be classified as either rational or irrational.

Key Concept

Rational Numbers

- **Words**: A rational number can be expressed as a ratio \( \frac{m}{n} \), where \( m \) and \( n \) are integers and \( n \) is not zero. The decimal form of a rational number is either a terminating or repeating decimal.
- **Examples**: \( \frac{1}{6}, 1.9, 2.575757..., -3, \sqrt{4}, 0 \)

Irrational Numbers

- **Words**: A real number that is not rational is irrational. The decimal form of an irrational number neither terminates nor repeats.
- **Examples**: \( \sqrt{5}, \pi, 0.010010001... \)

The sets of natural numbers, \( \{1, 2, 3, 4, 5, \ldots\} \), whole numbers, \( \{0, 1, 2, 3, 4, \ldots\} \), and integers, \( \{\ldots, -3, -2, -1, 0, 1, 2, \ldots\} \) are all subsets of the rational numbers. The whole numbers are a subset of the rational numbers because every whole number \( n \) is equal to \( \frac{n}{1} \).
The Venn diagram shows the relationships among these sets of numbers.

R = reals
Q = rationals
I = irrationals
Z = integers
W = wholes
N = naturals

The square root of any whole number is either a whole number or it is irrational. For example, \( \sqrt{36} \) is a whole number, but \( \sqrt{35} \), since it lies between 5 and 6, must be irrational.

**Example 1**

**Classify Numbers**

Name the sets of numbers to which each number belongs.

a. \( \sqrt{16} \)
   \[ \sqrt{16} = 4 \]
   naturals (N), wholes (W), integers (Z), rationals (Q), reals (R)

b. \(-185\)
   integers (Z), rationals (Q), and reals (R)

c. \( \sqrt{20} \)
   irrationals (I) and reals (R)
   \( \sqrt{20} \) lies between 4 and 5 so it is not a whole number.

d. \( \frac{-7}{8} \)
   rationals (Q) and reals (R)

e. 0.4\overline{5}
   rationals (Q) and reals (R)

The bar over the 45 indicates that those digits repeat forever.

**PROPERTIES OF REAL NUMBERS**

The real number system is an example of a mathematical structure called a field. Some of the properties of a field are summarized in the table below.

<table>
<thead>
<tr>
<th>Property</th>
<th>Addition</th>
<th>Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative</td>
<td>( a + b = b + a )</td>
<td>( a \cdot b = b \cdot a )</td>
</tr>
<tr>
<td>Associative</td>
<td>( (a + b) + c = a + (b + c) )</td>
<td>( (a \cdot b) \cdot c = a \cdot (b \cdot c) )</td>
</tr>
<tr>
<td>Identity</td>
<td>( a + 0 = a = 0 + a )</td>
<td>( a \cdot 1 = a = 1 \cdot a )</td>
</tr>
<tr>
<td>Inverse</td>
<td>( a + (-a) = 0 = (-a) + a )</td>
<td>If ( a \neq 0 ), then ( a \cdot \frac{1}{a} = 1 = \frac{1}{a} \cdot a ).</td>
</tr>
<tr>
<td>Distributive</td>
<td>( a(b + c) = ab + ac )  and ( (b + c)a = ba + ca )</td>
<td></td>
</tr>
</tbody>
</table>
**Example 2** Identify Properties of Real Numbers

Name the property illustrated by each equation.

a. \((5 + 7) + 8 = 8 + (5 + 7)\)

   Commutative Property of Addition

   The Commutative Property says that the order in which you add does not change the sum.

b. \(3(4x) = (3 \cdot 4)x\)

   Associative Property of Multiplication

   The Associative Property says that the way you group three numbers when multiplying does not change the product.

**Example 3** Additive and Multiplicative Inverses

Identify the additive inverse and multiplicative inverse for each number.

a. \(-1\frac{3}{4}\)

   Since \(-1\frac{3}{4} + (1\frac{3}{4}) = 0\), the additive inverse of \(-1\frac{3}{4}\) is \(1\frac{3}{4}\).

   Since \(-1\frac{3}{4} = -\frac{7}{4}\) and \((-\frac{7}{4})(-\frac{4}{7}) = 1\), the multiplicative inverse of \(-1\frac{3}{4}\) is \(-\frac{4}{7}\).

b. 1.25

   Since \(1.25 + (-1.25) = 0\), the additive inverse of 1.25 is \(-1.25\).

   The multiplicative inverse of 1.25 is \(\frac{1}{1.25}\) or 0.8.

   **CHECK** Notice that \(1.25 \times 0.8 = 1\).

You can model the Distributive Property using algebra tiles.

**Algebra Activity**

**Distributive Property**

- A 1 tile is a square that is 1 unit wide and 1 unit long. Its area is 1 square unit. An \(x\) tile is a rectangle that is 1 unit wide and \(x\) units long. Its area is \(x\) square units.

- To find the product \(3(x + 1)\), model a rectangle with a width of 3 and a length of \(x + 1\). Use your algebra tiles to mark off the dimensions on a product mat. Then make the rectangle with algebra tiles.

- The rectangle has \(3x\) tiles and 3 \(1\) tiles. The area of the rectangle is \(x + x + x + 1 + 1 + 1\) or \(3x + 3\). Thus, \(3(x + 1) = 3x + 3\).

**Model and Analyze**

Tell whether each statement is true or false. Justify your answer with algebra tiles and a drawing.

1. \(4(x + 2) = 4x + 2\)
2. \(3(2x + 4) = 6x + 7\)
3. \(2(3x + 5) = 6x + 10\)
4. \((4x + 1)5 = 4x + 5\)
The Distributive Property is often used in real-world applications.

**Example 4 Use the Distributive Property to Solve a Problem**

**FOOD SERVICE** A restaurant adds a 20% tip to the bills of parties of 6 or more people. Suppose a server waits on five such tables. The bill without the tip for each party is listed in the table. How much did the server make in tips during this shift?

<table>
<thead>
<tr>
<th>Party 1</th>
<th>Party 2</th>
<th>Party 3</th>
<th>Party 4</th>
<th>Party 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$185.45</td>
<td>$205.20</td>
<td>$195.05</td>
<td>$245.80</td>
<td>$262.00</td>
</tr>
</tbody>
</table>

There are two ways to find the total amount of tips received.

**Method 1**
Multiply each dollar amount by 20% or 0.2 and then add.

\[
T = 0.2(185.45) + 0.2(205.20) + 0.2(195.05) + 0.2(245.80) + 0.2(262)
\]
\[
= 37.09 + 41.04 + 39.01 + 49.16 + 52.40
\]
\[
= 218.70
\]

**Method 2**
Add the bills of all the parties and then multiply the total by 0.2.

\[
T = 0.2(185.45 + 205.20 + 195.05 + 245.80 + 262)
\]
\[
= 0.2(1093.50)
\]
\[
= 218.70
\]

The server made $218.70 during this shift.

Notice that both methods result in the same answer.

The properties of real numbers can be used to simplify algebraic expressions.

**Example 5 Simplify an Expression**

Simplify \(2(5m + n) + 3(2m - 4n)\).

\[
2(5m + n) + 3(2m - 4n)
\]
\[
= 2(5m) + 2(n) + 3(2m) - 3(4n) \quad \text{Distributive Property}
\]
\[
= 10m + 2n + 6m - 12n \quad \text{Multiply.}
\]
\[
= 10m + 6m + 2n - 12n \quad \text{Commutative Property (+)}
\]
\[
= (10 + 6)m + (2 - 12)n \quad \text{Distributive Property}
\]
\[
= 16m - 10n \quad \text{Simplify.}
\]

**Check for Understanding**

1. **Concept Check**
   - **OPEN ENDED** Give an example of each type of number.
     - a. natural
     - b. whole
     - c. integer
     - d. rational
     - e. irrational
     - f. real
   - **2. Explain** why \(\frac{\sqrt{3}}{2}\) is *not* a rational number.
   - **3. Disprove** the following statement by giving a counterexample. A **counterexample** is a specific case that shows that a statement is false. Explain.
     - Every real number has a multiplicative inverse.
Guided Practice

Name the sets of numbers to which each number belongs.
4. −4 5. 45 6. 6.23

Name the property illustrated by each equation.
7. \( \frac{2}{3} \cdot \frac{3}{2} = 1 \) 8. \((a + 4) + 2 = a + (4 + 2)\) 9. \(4x + 0 = 4x\)

Identify the additive inverse and multiplicative inverse for each number.
10. −8 11. \(\frac{1}{3}\) 12. 1.5

Simplify each expression.
13. \(3x + 4y - 5x\) 14. \(9p - 2n + 4p + 2n\)
15. \(3(5c + 4d) + 6(d - 2c)\) 16. \(\frac{1}{2}(16 - 4a) - \frac{3}{4}(12 + 20a)\)

Application

BAND BOOSTERS For Exercises 17 and 18, use the information below and in the table.
Ashley is selling chocolate bars for $1.50 each to raise money for the band.

17. Write an expression to represent the total amount of money Ashley raised during this week.
18. Evaluate the expression from Exercise 17 by using the Distributive Property.

Extra Practice
See page 828.

Practice and Apply

Name the sets of numbers to which each number belongs.
19. 0 20. \(-\frac{2}{9}\) 21. \(\sqrt{121}\) 22. \(-4.55\)
23. \(\sqrt{10}\) 24. \(-31\) 25. \(\frac{12}{2}\) 26. \(\frac{3\pi}{2}\)
27. Name the sets of numbers to which all of the following numbers belong. Then arrange the numbers in order from least to greatest.
   2.49, 2.49, 2.4, 2.49, 2.59

Name the property illustrated by each equation.
28. \(5a + (-5a) = 0\) 29. \((3 \cdot 4) \cdot 25 = 3 \cdot (4 \cdot 25)\)
30. \(-6xy + 0 = -6xy\) 31. \([5 + (-2)] + (-4) = 5 + [(-2) + (-4)]\)
32. \((2 + 14) + 3 = 3 + (2 + 14)\) 33. \(\left(\frac{12}{7}\right) \cdot \left(\frac{7}{9}\right) = 1\)
34. \(2\sqrt{3} + 5\sqrt{3} = (2 + 5)\sqrt{3}\) 35. \(ab = 1ab\)

NUMBER THEORY For Exercises 36–39, use the properties of real numbers to answer each question.
36. If \(m + n = m\), what is the value of \(n\)?
37. If \(m + n = 0\), what is the value of \(n\)? What is \(n\) called with respect to \(m\)?
38. If \(mn = 1\), what is the value of \(n\)? What is \(n\) called with respect to \(m\)?
39. If \(mn = m\), what is the value of \(n\)?

www.algebra2.com/self_check_quiz
For Exercises 40–42, use the following information.

The Greek mathematician Pythagoras believed that all things could be described by numbers. By “number” he meant positive integers.

40. To what set of numbers was Pythagoras referring when he spoke of “numbers?”

41. Use the formula \( c = \sqrt{2s^2} \) to calculate the length of the hypotenuse \( c \), or longest side, of this right triangle using \( s \), the length of one leg.

42. Explain why Pythagoras could not find a “number” to describe the value of \( c \).

Name the additive inverse and multiplicative inverse for each number.

43. \(-10\)
44. \(2.5\)
45. \(0.125\)
46. \(5\)
47. \(4\)
48. \(4\)

Simplify each expression.

49. \(7a + 3b - 4a - 5b\)
50. \(3x + 5y + 7x - 3y\)
51. \(3(15x - 9y) + 5(4y - x)\)
52. \(2(10m - 7a) + 3(8a - 3m)\)
53. \(8(y + 7t) - 4(13 + 5r)\)
54. \(4(14c - 10d) - 6(d + 4c)\)
55. \(4(0.2m - 0.3n) - 6(0.7m - 0.5n)\)
56. \(7(0.2p + 0.3q) + 5(0.6p - q)\)
57. \(\frac{1}{4}(6 + 20y) - \frac{1}{2}(19 - 8y)\)
58. \(\frac{1}{6}(3x + 5y) + \frac{2}{3}(\frac{3}{5}x - 6y)\)

Determine whether each statement is true or false. If false, give a counterexample.

59. Every whole number is an integer.
60. Every integer is a whole number.
61. Every real number is irrational.
62. Every integer is a rational number.

For Exercises 63 and 64, use the information below and in the table.

Andrea works as a hostess in a restaurant and is paid every two weeks.

63. If Andrea earns $6.50 an hour, illustrate the Distributive Property by writing two expressions representing Andrea’s pay last week.

64. Find the mean or average number of hours Andrea worked each day, to the nearest tenth of an hour. Then use this average to predict her pay for a two-week pay period.

WORK For Exercises 63 and 64, use the information below and in the table.

Andrea’s Hours

<table>
<thead>
<tr>
<th>Days of the week</th>
<th>Hours Worked</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>0.5</td>
</tr>
<tr>
<td>T</td>
<td>4.25</td>
</tr>
<tr>
<td>W</td>
<td>6.5</td>
</tr>
<tr>
<td>T</td>
<td>5.0</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
</tr>
<tr>
<td>S</td>
<td>4.25</td>
</tr>
<tr>
<td>S</td>
<td>5.0</td>
</tr>
</tbody>
</table>

65. BAKING Mitena is making two types of cookies. The first recipe calls for \(2\frac{1}{4}\) cups of flour, and the second calls for \(1\frac{1}{8}\) cups of flour. If Mitena wants to make 3 batches of the first recipe and 2 batches of the second recipe, how many cups of flour will she need? Use the properties of real numbers to show how Mitena could compute this amount mentally. Justify each step.
**BASKETBALL** For Exercises 66 and 67, use the diagram of an NCAA basketball court below.

![Diagram of an NCAA basketball court](image)

66. Illustrate the Distributive Property by writing two expressions for the area of the basketball court.

67. Evaluate the expression from Exercise 66 using the Distributive Property. What is the area of an NCAA basketball court?

**SCHOOL SHOPPING** For Exercises 68 and 69, use the graph at the right.

68. Illustrate the Distributive Property by writing two expressions to represent the amount that the average student spends shopping for school at specialty stores and department stores.

69. Evaluate the expression from Exercise 68 using the Distributive Property.

70. **CRITICAL THINKING** Is the Distributive Property also true for division? In other words, does \( \frac{b + c}{a} = \frac{b}{a} + \frac{c}{a}, a \neq 0 \)? If so, give an example and explain why it is true. If not true, give a counterexample.

71. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is the Distributive Property useful in calculating store savings?

Include the following in your answer:

- an explanation of how the Distributive Property could be used to calculate the coupon savings listed on a grocery receipt, and
- an example of how the Distributive Property could be used to calculate the savings from a clothing store sale where all items were discounted by the same percent.

72. If \( a \) and \( b \) are natural numbers, then which of the following must also be a natural number?

- **I.** \( a - b \)
  - A. I only
  - B. II only
  - C. I and II only
  - D. I and III only

- **II.** \( ab \)
  - A. I only
  - B. II only
  - C. I and II only
  - D. II and III only

- **III.** \( \frac{a}{b} \)
  - A. III only
  - B. IV only
  - C. I and III only
  - D. II and III only

73. If \( x = 1.4 \), find the value of 27(x + 1.2) - 26(x + 1.2).

- A. 1
- B. -0.4
- C. 2.6
- D. 65
For Exercises 74–77, use the following information.
The product of any two whole numbers is always a whole number. So, the set of whole numbers is said to be **closed** under multiplication. This is an example of the **Closure Property**. State whether each statement is **true** or **false**. If false, give a counterexample.

74. The set of integers is closed under multiplication.
75. The set of whole numbers is closed under subtraction.
76. The set of rational numbers is closed under addition.
77. The set of whole numbers is closed under division.

**Maintain Your Skills**

Find the value of each expression. (Lesson 1-1)

- 78. $9(4 - 3)^5$
- 79. $5 + 9 + 3(3) - 8$

Evaluate each expression if $a = -5$, $b = 0.25$, $c = \frac{1}{2}$, and $d = 4$. (Lesson 1-1)

- 80. $a + 2b - c$
- 81. $b + 3(a + d)^3$

82. **GEOMETRY** The formula for the surface area $SA$ of a rectangular prism is $SA = 2\ell w + 2\ell h + 2wh$, where $\ell$ represents the length, $w$ represents the width, and $h$ represents the height. Find the surface area of the rectangular prism. (Lesson 1-1)

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Evaluate each expression if $a = 2$, $b = -\frac{3}{4}$, and $c = 1.8$. (To review evaluating expressions, see Lesson 1-1.)

- 83. $8b - 5$
- 84. $\frac{2}{5}b + 1$
- 85. $1.5c - 7$
- 86. $-9(a - 6)$

**Practice Quiz 1**

**Lessons 1-1 and 1-2**

Find the value of each expression. (Lesson 1-1)

1. $18 - 12 ÷ 3$
2. $-4 + 5(7 - 2^3)$
3. $\frac{18 + 3 \times 4}{13 - 8}$

4. Evaluate $a^3 + b(c - d)$ if $a = -2$, $b = \frac{1}{3}$, and $c = -12$. (Lesson 1-1)

5. **ELECTRICITY** Find the amount of current $I$ (in amperes) produced if the electromotive force $E$ is 2.5 volts, the circuit resistance $R$ is 1.05 ohms, and the resistance $r$ within a battery is 0.2 ohm. Use the formula $I = \frac{E}{R + r}$. (Lesson 1-1)

Name the sets of numbers to which each number belongs. (Lesson 1-2)

- 6. 3.5
- 7. $\sqrt{100}$
- 8. Name the property illustrated by $bc + (-bc) = 0$. (Lesson 1-2)
- 9. Name the additive inverse and multiplicative inverse of $\frac{6}{7}$. (Lesson 1-2)
- 10. Simplify $4(14x - 10y) - 6(x + 4y)$. (Lesson 1-2)
Investigating Polygons and Patterns

Collect the Data
Use a ruler or geometry drawing software to draw six large polygons with 3, 4, 5, 6, 7, and 8 sides. The polygons do not need to be regular. Convex polygons, ones whose diagonals lie in the interior, will be best for this activity.

1. Copy the table below and complete the column labeled Diagonals by drawing the diagonals for all six polygons and record your results.

<table>
<thead>
<tr>
<th>Figure Name</th>
<th>Sides ((n))</th>
<th>Diagonals</th>
<th>Diagonals From One Vertex</th>
</tr>
</thead>
<tbody>
<tr>
<td>triangle</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>quadrilateral</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>pentagon</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>hexagon</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>heptagon</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>octagon</td>
<td>8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Analyze the Data
2. Describe the pattern shown by the number of diagonals in the table above.
3. Complete the last column in the table above by recording the number of diagonals that can be drawn from one vertex of each polygon.
4. Write an expression in terms of \(n\) that relates the number of diagonals from one vertex to the number of sides for each polygon.
5. If a polygon has \(n\) sides, how many vertices does it have?
6. How many vertices does one diagonal connect?

Make a Conjecture
7. Write a formula in terms of \(n\) for the number of diagonals of a polygon of \(n\) sides. \((Hint:\ Consider your answers to Exercises 2, 3, and 4.\)
8. Draw a polygon with 10 sides. Test your formula for the decagon.
9. Explain how your formula relates to the number of vertices of the polygon and the number of diagonals that can be drawn from each vertex.

Extend the Activity
10. Draw 3 noncollinear dots on your paper. Determine the number of lines that are needed to connect each dot to every other dot. Continue by drawing 4 dots, 5 dots, and so on and finding the number of lines to connect them.
11. Copy and complete the table at the right.
12. Use any method to find a formula that relates the number of dots, \(x\), to the number of lines, \(y\).
13. Explain why the formula works.
Vocabulary
- open sentence
- equation
- solution

**VERBAL EXPRESSIONS TO ALGEBRAIC EXPRESSIONS** Verbal expressions can be translated into algebraic or mathematical expressions using the language of algebra. Any letter can be used as a variable to represent a number that is not known.

**Example 1** Verbal to Algebraic Expression

Write an algebraic expression to represent each verbal expression.

a. 7 less than a number \( n - 7 \)

b. three times the square of a number \( 3x^2 \)

c. the cube of a number increased by 4 times the same number \( p^3 + 4p \)

d. twice the sum of a number and 5 \( 2(y + 5) \)

A mathematical sentence containing one or more variables is called an open sentence. A mathematical sentence stating that two mathematical expressions are equal is called an equation.

**Example 2** Algebraic to Verbal Sentence

Write a verbal sentence to represent each equation.

a. \( 10 = 12 - 2 \) Ten is equal to 12 minus 2.

b. \( n + (-8) = -9 \) The sum of a number and \(-8\) is \(-9\).

c. \( \frac{n}{6} = n^2 \) A number divided by 6 is equal to that number squared.

Open sentences are neither true nor false until the variables have been replaced by numbers. Each replacement that results in a true sentence is called a solution of the open sentence.
PROPERTIES OF EQUALITY  To solve equations, we can use properties of equality. Some of these equivalence relations are listed in the table below.

<table>
<thead>
<tr>
<th><strong>Key Concept</strong></th>
<th><strong>Properties of Equality</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Property</strong></td>
<td><strong>Symbols</strong></td>
</tr>
<tr>
<td>Reflexive</td>
<td>For any real number $a$, $a = a$.</td>
</tr>
<tr>
<td>Symmetric</td>
<td>For all real numbers $a$ and $b$, if $a = b$, then $b = a$.</td>
</tr>
<tr>
<td>Transitive</td>
<td>For all real numbers $a$, $b$, and $c$, if $a = b$ and $b = c$, then $a = c$.</td>
</tr>
<tr>
<td>Substitution</td>
<td>If $a = b$, then $a$ may be replaced by $b$ and $b$ may be replaced by $a$.</td>
</tr>
</tbody>
</table>

**Example 3  Identify Properties of Equality**

Name the property illustrated by each statement.

a. If $3m = 5n$ and $5n = 10p$, then $3m = 10p$.
   Transitive Property of Equality

b. If $-11a + 2 = -3a$, then $-3a = -11a + 2$.
   Symmetric Property of Equality

Sometimes an equation can be solved by adding the same number to each side or by subtracting the same number from each side or by multiplying or dividing each side by the same number.

**Example 4  Solve One-Step Equations**

Solve each equation. Check your solution.

a. $a + 4.39 = 76$
   $a + 4.39 = 76$  Original equation
   $a = 71.61$  Simplify.
   The solution is 71.61.

(continued on the next page)
**Study Tip**

**Multiplication and Division Properties of Equality**

Example 4b could also have been solved using the Division Property of Equality. Note that dividing each side of the equation by \( \frac{5}{3} \) is the same as multiplying each side by \( \frac{3}{5} \).

CHECK

a. \( a + 4.39 = 76 \)
   
   \[
   \begin{align*}
   71.61 + 4.39 & \leq 76 \\
   76 & = 76 \checkmark
   \end{align*}
   \]
   
   Substitute 71.61 for \( a \).

b. \( -\frac{3}{5} \cdot d = 18 \)

\[
\begin{align*}
-\frac{3}{5} \cdot d &= 18 & \text{Original equation} \\
\frac{5}{3} \left( -\frac{3}{5} \right) d &= \frac{5}{3} \cdot 18 & \text{Multiply each side by} \ -\frac{5}{3}, \text{the multiplicative inverse of} \ -\frac{3}{5}. \\
d &= -30 & \text{Simplify.}
\end{align*}
\]

The solution is \( -30 \).

CHECK

\[
\begin{align*}
-\frac{3}{5} \cdot (-30) & \leq 18 & \text{Substitute} \ -30 \ \text{for} \ d. \\
18 & = 18 \checkmark
\end{align*}
\]

Sometimes you must apply more than one property to solve an equation.

**Example 5** **Solve a Multi-Step Equation**

Solve \( 2(2x + 3) - 3(4x - 5) = 22 \).

\[
\begin{align*}
2(2x + 3) - 3(4x - 5) &= 22 & \text{Original equation} \\
4x + 6 - 12x + 15 &= 22 & \text{Distributive and Substitution Properties} \\
-8x + 21 &= 22 & \text{Commutative, Distributive, and Substitution Properties} \\
-8x &= 1 & \text{Subtraction and Substitution Properties} \\
x &= -\frac{1}{8} & \text{Division and Substitution Properties}
\end{align*}
\]

The solution is \( -\frac{1}{8} \).

You can use properties of equality to solve an equation or formula for a specified variable.

**Example 6** **Solve for a Variable**

**GEOMETRY** The surface area of a cone is \( S = \pi r \ell + \pi r^2 \), where \( S \) is the surface area, \( \ell \) is the slant height of the cone, and \( r \) is the radius of the base. Solve the formula for \( \ell \).

\[
\begin{align*}
S &= \pi r \ell + \pi r^2 & \text{Surface area formula} \\
S - \pi r^2 &= \pi r \ell + \pi r^2 - \pi r^2 & \text{Subtract} \ \pi r^2 \ \text{from each side.} \\
S - \pi r^2 &= \pi r \ell & \text{Simplify.} \\
\frac{S - \pi r^2}{\pi r} &= \ell & \text{Divide each side by} \ \pi r. \\
\end{align*}
\]
Many standardized test questions can be solved by using properties of equality.

**Example 7**  
**Apply Properties of Equality**

Multiple-Choice Test Item

If \(3n - 8 = \frac{9}{5}\), what is the value of \(3n - 3\)?

- A \(\frac{34}{5}\)
- B \(\frac{49}{15}\)
- C \(-\frac{16}{5}\)
- D \(-\frac{27}{5}\)

**Read the Test Item**

You are asked to find the value of the expression \(3n - 3\). Your first thought might be to find the value of \(n\) and then evaluate the expression using this value. Notice, however, that you are not required to find the value of \(n\). Instead, you can use the Addition Property of Equality on the given equation to find the value of \(3n - 3\).

**Solve the Test Item**

\[
\begin{align*}
3n - 8 &= \frac{9}{5} & \text{Original equation} \\
3n - 8 + 5 &= \frac{9}{5} + 5 & \text{Add } 5 \text{ to each side.} \\
3n - 3 &= \frac{34}{5} & \frac{9}{5} + 5 = \frac{9}{5} + \frac{25}{5} \text{ or } \frac{34}{5}
\end{align*}
\]

The answer is A.

To solve a word problem, it is often necessary to define a variable and write an equation. Then solve by applying the properties of equality.

**Example 8**  
**Write an Equation**

**HOME IMPROVEMENT**  
Josh and Pam have bought an older home that needs some repair. After budgeting a total of \$1685 for home improvements, they started by spending \$425 on small improvements. They would like to replace six interior doors next. What is the maximum amount they can afford to spend on each door?

**Explore**  
Let \(c\) represent the cost to replace each door.

**Plan**  
Write and solve an equation to find the value of \(c\).

**Solve**

\[
\begin{align*}
\text{The number of doors} & \times \text{the cost to replace each door} + \text{previous expenses} & = \text{the total cost.} \\
6 & \cdot c + 425 & = 1685 \\
6c + 425 - 425 &= 1685 - 425 & \text{Subtract } 425 \text{ from each side.} \\
6c &= 1260 & \text{Simplify.} \\
\frac{6c}{6} &= \frac{1260}{6} & \text{Divide each side by } 6. \\
c &= 210 & \text{Simplify.}
\end{align*}
\]

They can afford to spend \$210 on each door.

**Examine**  
The total cost to replace six doors at \$210 each is \(6(210)\) or \$1260. Add the other expenses of \$425 to that, and the total home improvement bill is \$1260 + 425 or \$1685. Thus, the answer is correct.
1. OPEN ENDED Write an equation whose solution is \(-7\).

2. Determine whether the following statement is sometimes, always, or never true. Explain.
   
   Dividing each side of an equation by the same expression produces an equivalent equation.

3. FIND THE ERROR Crystal and Jamal are solving \(C = \frac{5}{9}(F - 32)\) for \(F\).

   \[
   \text{Crystal:} \quad C = \frac{5}{9}(F - 32) \\
   C + 32 = \frac{5}{9}F \\
   \frac{9}{5}(C + 32) = F
   \]

   \[
   \text{Jamal:} \quad C = \frac{5}{9}(F - 32) \\
   \frac{9}{5}C = F - 32 \\
   \frac{9}{5}C + 32 = F
   \]

   Who is correct? Explain your reasoning.

Guided Practice

Write an algebraic expression to represent each verbal expression.

4. five increased by four times a number
5. twice a number decreased by the cube of the same number

Write a verbal expression to represent each equation.

6. \(9n - 3 = 6\) 
7. \(5 + 3x^2 = 2x\)

Name the property illustrated by each statement.

8. \((3x + 2) - 5 = (3x + 2) - 5\) 
9. If \(4c = 15\), then \(4c + 2 = 15 + 2\)

Solve each equation. Check your solution.

10. \(y + 14 = -7\) 
11. \(7 + 3x = 49\) 
12. \(-4(b + 7) = -12\)
13. \(7q + q - 3q = -24\) 
14. \(1.8a - 5 = -2.3\) 
15. \(-\frac{3}{4}n + 1 = -11\)

Solve each equation or formula for the specified variable.

16. \(4y - 2n = 9\), for \(y\) 
17. \(I = prt\), for \(p\)

18. If \(4x + 7 = 18\), what is the value of \(12x + 21\)?

   A 2.75 B 32 C 33 D 54

Practice and Apply

Write an algebraic expression to represent each verbal expression.

19. the sum of 5 and three times a number
20. seven more than the product of a number and 10
21. four less than the square of a number
22. the product of the cube of a number and \(-6\)
23. five times the sum of 9 and a number
24. twice the sum of a number and 8
25. the square of the quotient of a number and 4
26. the cube of the difference of a number and 7
GEOMETRY  For Exercises 27 and 28, use the following information.
The formula for the surface area of a cylinder with radius \( r \) and height \( h \) is \( \pi r^2 \times h + 2 \pi rh \) plus twice the product of \( \pi \) and the square of the radius.

27. Translate this verbal expression of the formula into an algebraic expression.
28. Write an equivalent expression using the Distributive Property.

Write a verbal expression to represent each equation.

29. \( x - 5 = 12 \)  
30. \( 2n + 3 = -1 \)  
31. \( y^2 = 4y \)  
32. \( 3a^3 = a + 4 \)  
33. \( \frac{b}{4} = 2(b + 1) \)  
34. \( 7 - \frac{1}{2}x = \frac{3}{x^2} \)

Name the property illustrated by each statement.
35. If \( 3(-2)z = 24 \), then \(-6z = 24\).
36. If \( 5 + b = 13 \), then \( b = 8 \).
37. If \( 2x = 3d \) and \( 3d = -4 \), then \( 2x = -4 \).
38. If \( g - t = n \), then \( g = n + t \).
39. If \( 14 = \frac{x}{2} + 11 \), then \( \frac{x}{2} + 11 = 14 \).
40. If \( y - 2 = -8 \), then \( 3(y - 2) = 3(-8) \).

Solve each equation. Check your solution.

41. \( 2p + 15 = 29 \)  
42. \( 14 - 3n = -10 \)  
43. \( 7a - 3a + 2a - a = 16 \)  
44. \( x + 9x - 6x + 4x = 20 \)  
45. \( \frac{1}{9} - \frac{2}{3}b = \frac{1}{18} \)  
46. \( 5 \times \frac{3}{4}x = \frac{1}{16} \)  
47. \( 27 = -9(y + 5) \)  
48. \( -7(p + 8) = 21 \)  
49. \( 3f - 2 = 4f + 5 \)  
50. \( 3d + 7 = 6d + 5 \)  
51. \( 4.3n + 1 = 7 - 1.7n \)  
52. \( 1.7x - 8 = 2.7x + 4 \)  
53. \( 3(2z + 25) - 2(z - 1) = 78 \)  
54. \( 4(k + 3) + 2 = 4.5(k + 1) \)  
55. \( \frac{3}{11}a - 1 = \frac{7}{11}a + 9 \)  
56. \( \frac{2}{5}x + \frac{3}{7} = 1 - \frac{4}{7}x \)

Solve each equation or formula for the specified variable.

57. \( d = rt \), for \( r \)  
58. \( x = \frac{-b}{2a} \), for \( a \)  
59. \( V = \frac{1}{3} \pi r^2h \), for \( h \)  
60. \( A = \frac{1}{2}h(a + b) \), for \( b \)  
61. \( \frac{a(b - 2)}{c - 3} = x \), for \( b \)  
62. \( x = \frac{y}{y + 4} \), for \( y \)

Define a variable, write an equation, and solve the problem.

63. **Bowling**  Jon and Morgan arrive at Sunnybrook Lanes with $16.75. Find the maximum number of games they can bowl if they each rent shoes.
For Exercises 64–70, define a variable, write an equation, and solve the problem.

64. **GEOMETRY** The perimeter of a regular octagon is 124 inches. Find the length of each side.

65. **CAR EXPENSES** Benito spent $1837 to operate his car last year. Some of these expenses are listed below. Benito’s only other expense was for gasoline. If he drove 7600 miles, what was the average cost of the gasoline per mile?

<table>
<thead>
<tr>
<th>Operating Expenses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insurance: $972</td>
</tr>
<tr>
<td>Registration: $114</td>
</tr>
<tr>
<td>Maintenance: $105</td>
</tr>
</tbody>
</table>

66. **SCHOOL** A school conference room can seat a maximum of 83 people. The principal and two counselors need to meet with the school’s student athletes to discuss eligibility requirements. If each student must bring a parent with them, what is the maximum number of students that can attend each meeting?

67. **FAMILY** Chun-Wei’s mother is 8 more than twice his age. His father is three years older than his mother is. If all three family members have lived 94 years, how old is each family member?

68. **SCHOOL TRIP** The Parent Teacher Organization has raised $1800 to help pay for a trip to an amusement park. They ask that there be one adult for every five students attending. Adult tickets cost $45 and student tickets cost $30. If the group wants to take 50 students, how much will each student need to pay so that adults agreeing to chaperone pay nothing?

69. **BUSINESS** A trucking company is hired to deliver 125 lamps for $12 each. The company agrees to pay $45 for each lamp that is broken during transport. If the trucking company needs to receive a minimum payment of $1364 for the shipment to cover their expenses, find the maximum number of lamps they can afford to break during the trip.

70. **PACKAGING** Two designs for a soup can are shown at the right. If each can holds the same amount of soup, what is the height of can A?

71. **RAILROADS** For Exercises 71–73, use the following information.

The First Transcontinental Railroad was built by two companies. The Central Pacific began building eastward from Sacramento, California, while the Union Pacific built westward from Omaha, Nebraska. The two lines met at Promontory, Utah, in 1869, about 6 years after construction began.

71. The Central Pacific Company laid an average of 9.6 miles of track per month. Together the two companies laid a total of 1775 miles of track. Determine the average number of miles of track laid per month by the Union Pacific Company.

72. About how many miles of track did each company lay?

73. Why do you think the Union Pacific was able to lay track so much more quickly than the Central Pacific?
74. **MONEY**  Allison is saving money to buy a video game system. In the first week, her savings were $8 less than \( \frac{2}{5} \) the price of the system. In the second week, she saved 50 cents more than \( \frac{1}{2} \) the price of the system. She was still $37 short. Find the price of the system.

75. **CRITICAL THINKING**  Write a verbal expression to represent the algebraic expression \( 3(x - 5) + 4x(x + 1) \).

76. **WRITING IN MATH**  Answer the question that was posed at the beginning of the lesson.

*How can you find the most effective level of intensity for your workout?*

Include the following in your answer:

• an explanation of how to find the age of a person who is exercising at an 80% level of intensity \( I \) with a pulse count of 27, and

• a description of when it would be desirable to solve a formula like the one given for a specified variable.

77. If \( -6x + 10 = 17 \), then \( 3x - 5 = \)

\[\begin{align*}
\text{A} & : \frac{-7}{6} \\
\text{B} & : \frac{-17}{2} \\
\text{C} & : 2. \\
\text{D} & : \frac{19}{3} \\
\text{E} & : \frac{5}{3}.
\end{align*}\]

78. In triangle \( PQR \), \( QS \) and \( SR \) are angle bisectors and angle \( P = 74^\circ \). How many degrees are there in angle \( QSR \)?

\[\begin{align*}
\text{A} & : 106 \\
\text{B} & : 121 \\
\text{C} & : 125 \\
\text{D} & : 127 \\
\text{E} & : 143.
\end{align*}\]

79. **Mixed Review**  Simplify each expression.  

\[2x + 9y + 4z - y - 8x\]

\[4(2a + 5b) - 3(4b - a)\]

80. Evaluate each expression if \( a = 3 \), \( b = -2 \), and \( c = 1.2 \).

\[a - [b(a - c)]\]

\[c^2 - ab\]

81. **GEOMETRY**  The formula for the surface area \( S \) of a regular pyramid is \( S = \frac{1}{2}P\ell + B \), where \( P \) is the perimeter of the base, \( \ell \) is the slant height, and \( B \) is the area of the base. Find the surface area of the square-based pyramid shown at the right.

82. **PREREQUISITE SKILL**  Identify the additive inverse for each number or expression.

\[5\]

\[-3\]

\[2.5\]

\[\frac{1}{4}\]

\[-3x\]

\[5 - 6y\]
Vocabulary
• absolute value
• empty set

Solving Absolute Value Equations

What You’ll Learn
• Evaluate expressions involving absolute values.
• Solve absolute value equations.

How can an absolute value equation describe the magnitude of an earthquake?

Seismologists use the Richter scale to express the magnitudes of earthquakes. This scale ranges from 1 to 10, 10 being the highest. The uncertainty in the estimate of a magnitude \( E \) is about plus or minus 0.3 unit. This means that an earthquake with a magnitude estimated at 6.1 on the Richter scale might actually have a magnitude as low as 5.8 or as high as 6.4. These extremes can be described by the absolute value equation \( | E - 6.1 | = 0.3 \).

Absolute Value Expressions

The absolute value of a number is its distance from 0 on the number line. Since distance is nonnegative, the absolute value of a number is always nonnegative. The symbol \( | x | \) is used to represent the absolute value of a number \( x \).

Key Concept

Absolute Value

- Words: For any real number \( a \), if \( a \) is positive or zero, the absolute value of \( a \) is \( a \). If \( a \) is negative, the absolute value of \( a \) is the opposite of \( a \).
- Symbols: For any real number \( a \), \( | a | = a \) if \( a \geq 0 \), and \( | a | = -a \) if \( a < 0 \).
- Model: \( | -3 | = 3 \) and \( | 3 | = 3 \)

When evaluating expressions that contain absolute values, the absolute value bars act as a grouping symbol. Perform any operations inside the absolute value bars first.

Example 1 Evaluate an Expression with Absolute Value

Evaluate \( 1.4 + | 5y - 7 | \) if \( y = -3 \).

\[
1.4 + | 5y - 7 | = 1.4 + | 5(-3) - 7 | \\
= 1.4 + | -15 - 7 | \\
= 1.4 + | -22 | \\
= 1.4 + 22 \\
= 23.4
\]

The value is 23.4.
ABSOLUTE VALUE EQUATIONS  Some equations contain absolute value expressions. The definition of absolute value is used in solving these equations. For any real numbers \(a\) and \(b\), where \(b \geq 0\), if \(|a| = b\), then \(a = b\) or \(-a = b\). This second case is often written as \(a = -b\).

**Example 2  Solve an Absolute Value Equation**

Solve \(|x - 18| = 5\). Check your solutions.

Case 1  \(a = b\) or Case 2  \(a = -b\)

\[
\begin{align*}
\text{Case 1} & : x - 18 = 5 \\
x - 18 + 18 & = 5 + 18 \\
x & = 23
\end{align*}
\]

\[
\begin{align*}
\text{Case 2} & : x - 18 = -5 \\
x - 18 + 18 & = -5 + 18 \\
x & = 13
\end{align*}
\]

**CHECK**

\[
\begin{align*}
|x - 18| & = 5 \\
|23 - 18| & \neq 5 \\
|5| & \neq 5 \\
5 & = 5 \checkmark
\end{align*}
\]

The solutions are 23 or 13. Thus, the solution set is \(\{13, 23\}\).

On the number line, we can see that each answer is 5 units away from 18.

Because the absolute value of a number is always positive or zero, an equation like \(|x| = -5\) is never true. Thus, it has no solution. The solution set for this type of equation is the **empty set**, symbolized by \(\emptyset\) or \(\{\}\).

**Example 3  No Solution**

Solve \(|5x - 6| + 9 = 0\).

\[
\begin{align*}
|5x - 6| + 9 & = 0 \\
|5x - 6| & = -9
\end{align*}
\]

This sentence is **never** true. So the solution set is \(\emptyset\).

It is important to check your answers when solving absolute value equations. Even if the correct procedure for solving the equation is used, the answers may not be actual solutions of the original equation.

**Example 4  One Solution**

Solve \(|x + 6| = 3x - 2\). Check your solutions.

Case 1  \(a = b\) or Case 2  \(a = -b\)

\[
\begin{align*}
\text{Case 1} & : x + 6 = 3x - 2 \\
x + 6 & = 3x - 2 \\
6 & = 2x \\
8 & = 2x \\
4 & = x
\end{align*}
\]

\[
\begin{align*}
\text{Case 2} & : x + 6 = -(3x - 2) \\
x + 6 & = -3x + 2 \\
4x & = 4 \\
4 & = x
\end{align*}
\]

There appear to be two solutions, 4 or \(-1\).
CHECK \[ |x + 6| = 3x - 2 \quad |x + 6| = 3x - 2 \]
 \[ |4 + 6| = 3(4) - 2 \quad \text{or} \quad |-1 + 6| = 3(-1) - 2 \]
 \[ |10| = 12 - 2 \quad |5| = -3 - 2 \]
 \[ 10 = 10 \checkmark \]

Since 5 \(\neq\) -5, the only solution is 4. Thus, the solution set is \{4\}.

**Check for Understanding**

**Concept Check**

1. Explain why if the absolute value of a number is always nonnegative, \(|a|\) can equal \(-a\).

2. Write an absolute value equation for each solution set graphed below.

   - a.
     \[ \begin{array}{c}
     \text{a.} \\
     \text{4 units} \\
     \text{5} \\
     \text{5} \\
     \text{10 units} \\
     \text{0} \\
     \text{5} \\
     \text{a.} \\
     \text{4 units} \\
     \text{5} \\
     \text{5} \\
     \text{10 units} \\
     \text{0} \\
     \text{5} \\
     \end{array} \]

   - b.
     \[ \begin{array}{c}
     \text{b.} \\
     \text{2 units} \\
     \text{1} \\
     \text{5} \\
     \text{3 units} \\
     \text{6} \\
     \text{b.} \\
     \text{2 units} \\
     \text{1} \\
     \text{5} \\
     \text{3 units} \\
     \text{6} \\
     \end{array} \]

3. Determine whether the following statement is sometimes, always, or never true. Explain.
   For all real numbers \(a\) and \(b\), \(a \neq 0\), the equation \(|ax + b| = 0\) will have one solution.

4. **OPEN ENDED** Write and evaluate an expression with absolute value.

**Guided Practice**

Evaluate each expression if \(a = -4\) and \(b = 1.5\).

5. \(|a + 12|\)

6. \(|-6b|\)

7. \(-|a + 21|\)

Solve each equation. Check your solutions.

8. \(|x + 4| = 17\)

9. \(|b + 15| = 3\)

10. \(|a - 9| = 20\)

11. \(|y - 2| = 34\)

12. \(|2w + 3| + 6 = 2\)

13. \(|c - 2| = 2c - 10\)

**Application**

**FOOD** For Exercises 14–16, use the following information.

A meat thermometer is used to assure that a safe temperature has been reached to destroy bacteria. Most meat thermometers are accurate to within plus or minus 2\(^\circ\)F. **Source:** U.S. Department of Agriculture

14. The ham you are baking needs to reach an internal temperature of 160\(^\circ\)F. If the thermometer reads 160\(^\circ\)F, write an equation to determine the least and greatest temperatures of the meat.

15. Solve the equation you wrote in Exercise 14.

16. To what temperature reading should you bake a ham to ensure that the minimum internal temperature is reached? Explain.

**Practice and Apply**

Evaluate each expression if \(a = -5\), \(b = 6\), and \(c = 2.8\).

17. \(|-3a|\)

18. \(|-4b|\)

19. \(|a + 5|\)

20. \(|2 - b|\)

21. \(|2b - 15|\)

22. \(|4a + 7|\)

23. \(|-18 - 5c|\)

24. \(|c - a|\)

25. \(|6 - |3c + 7||\)

26. \(|9 - |-2b + 8||\)

27. \(|3|a - 10| + |2a|\)

28. \(|a - b| - |10c - a|\)
Solve each equation. Check your solutions.

29. \(|x - 25| = 17\)  
30. \(|y + 9| = 21\)

31. \(|a + 12| = 33\)  
32. \(2|b + 4| = 48\)

33. \(8|w - 7| = 72\)  
34. \(3|x + 5| = 11\)

35. \(|2z - 3| = 0\)  
36. \(|6c - 1| = -2\)

37. \(7|4x - 13| = 35\)  
38. \(-3|2n + 5| = -9\)

39. \(-12|9x + 1| = 144\)  
40. \(|5x + 9| + 6 = 1\)

41. \(|a - 3| - 14 = -6\)  
42. \(3|p - 5| = 2p\)

43. \(3|2a + 7| = 3a + 12\)  
44. \(|3x - 7| - 5 = -3\)

45. \(4|3t + 8| = 16t\)  
46. \(|15 + m| = -2m + 3\)

47. **COFFEE** Some say that to brew an excellent cup of coffee, you must have a brewing temperature of 200°F, plus or minus five degrees. Write and solve an equation describing the maximum and minimum brewing temperatures for an excellent cup of coffee.

48. **MANUFACTURING** A machine is used to fill each of several bags with 16 ounces of sugar. After the bags are filled, another machine weighs them. If the bag weighs 0.3 ounce more or less than the desired weight, the bag is rejected. Write an equation to find the heaviest and lightest bag the machine will approve.

49. **METEOROLOGY** The atmosphere of Earth is divided into four layers based on temperature variations. The troposphere is the layer closest to the planet. The average upper boundary of the layer is about 13 kilometers above Earth’s surface. This height varies with latitude and with the seasons by as much as 5 kilometers. Write and solve an equation describing the maximum and minimum heights of the upper bound of the troposphere.

**CRITICAL THINKING** For Exercises 50 and 51, determine whether each statement is sometimes, always, or never true. Explain your reasoning.

50. If \(a\) and \(b\) are real numbers, then \(|a + b| = |a| + |b|\).

51. If \(a, b,\) and \(c\) are real numbers, then \(c|a + b| = |ca + cb|\).

52. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can an absolute value equation describe the magnitude of an earthquake?

Include the following in your answer:

- a verbal and graphical explanation of how \(|E - 6.1| = 0.3\) describes the possible extremes in the variation of the earthquake’s magnitude, and
- an equation to describe the extremes for a different magnitude.

53. Which of the graphs below represents the solution set for \(|x - 3| - 4 = 0\)?

A) ![Graph A]  
B) ![Graph B]  
C) ![Graph C]  
D) ![Graph D]
54. Find the value of $-| -9 | - | 4 | - 3 | 5 - 7 |$.
   (A) $-19$  (B) $-11$  (C) $-7$  (D) $11$

For Exercises 55–58, consider the equation $| x + 1 | + 2 = | x + 4 |$.

55. To solve this equation, we must consider the case where $x + 4 \geq 0$ and the case where $x + 4 < 0$. Write the equations for each of these cases.

56. Notice that each equation you wrote in Exercise 55 has two cases. For each equation, write two other equations taking into consideration the case where $x + 1 \geq 0$ and the case where $x + 1 < 0$.

57. Solve each equation you wrote in Exercise 56. Then, check each solution in the original equation, $| x + 1 | + 2 = | x + 4 |$. What are the solution(s) to this absolute value equation?

58. **MAKE A CONJECTURE** For equations with one set of absolute value symbols, two cases must be considered. For an equation with two sets of absolute value symbols, four cases must be considered. How many cases must be considered for an equation containing three sets of absolute value symbols?

---

**Maintain Your Skills**

**Mixed Review**

Write an algebraic expression to represent each verbal expression. (Lesson 1-3)

59. twice the difference of a number and 11
60. the product of the square of a number and 5

Solve each equation. Check your solution. (Lesson 1-3)

61. $3x + 6 = 22$  
62. $7p - 4 = 3(4 + 5p)$  
63. $\frac{5}{7}y - 3 = \frac{3}{7}y + 1$

Name the property illustrated by each equation. (Lesson 1-2)

64. $(5 + 9) + 13 = 13 + (5 + 9)$  
65. $m(4 - 3) = m \cdot 4 - m \cdot 3$
66. $\left( \frac{1}{4} \right)^4 = 1$  
67. $5x + 0 = 5x$

Determine whether each statement is true or false. If false, give a counterexample. (Lesson 1-2)

68. Every real number is a rational number.
69. Every natural number is an integer.
70. Every irrational number is a real number.
71. Every rational number is an integer.

**GEOMETRY** For Exercises 72 and 73, use the following information.

The formula for the area $A$ of a triangle is $A = \frac{1}{2}bh$, where $b$ is the measure of the base and $h$ is the measure of the height. (Lesson 1-1)

72. Write an expression to represent the area of the triangle above.
73. Evaluate the expression you wrote in Exercise 72 for $x = 23$.

---

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Solve each equation. (To review solving equations, see page 20.)

74. $14y - 3 = 25$
75. $4.2x + 6.4 = 40$
76. $7w + 2 = 3w - 6$
77. $2(a - 1) = 8a - 6$
78. $48 + 5y = 96 - 3y$
79. $\frac{2x + 3}{5} = \frac{3}{10}$
**What You’ll Learn**

- Solve inequalities.
- Solve real-world problems involving inequalities.

**Vocabulary**

- set-builder notation
- interval notation

**How can inequalities be used to compare phone plans?**

Kuni is trying to decide between two rate plans offered by a wireless phone company.

<table>
<thead>
<tr>
<th>Plan 1</th>
<th>Plan 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly Access Fee</td>
<td>$35.00</td>
</tr>
<tr>
<td>Minutes Included</td>
<td>150</td>
</tr>
<tr>
<td>Additional Minutes</td>
<td>40¢</td>
</tr>
</tbody>
</table>

To compare these two rate plans, we can use inequalities. The monthly access fee for Plan 1 is less than the fee for Plan 2, \(35 < 55\). However, the additional minutes fee for Plan 1 is greater than that of Plan 2, \(0.40 > 0.35\).

**SOLVE INEQUALITIES**

For any two real numbers, \(a\) and \(b\), exactly one of the following statements is true.

\[ a < b \quad a = b \quad a > b \]

This is known as the **Trichotomy Property** or the **property of order**.

Adding the same number to, or subtracting the same number from, each side of an inequality does not change the truth of the inequality.

**Key Concept**

**Addition Property of Inequality**

- **Words**
  - For any real numbers, \(a\), \(b\), and \(c\):
    - If \(a > b\), then \(a + c > b + c\).
    - If \(a < b\), then \(a + c < b + c\).

- **Example**
  - \(3 < 5\)
  - \(3 + (-4) < 5 + (-4)\)
  - \(-1 < 1\)

**Subtraction Property of Inequality**

- **Words**
  - For any real numbers, \(a\), \(b\), and \(c\):
    - If \(a > b\), then \(a - c > b - c\).
    - If \(a < b\), then \(a - c < b - c\).

- **Example**
  - \(2 > -7\)
  - \(2 - 8 > -7 - 8\)
  - \(-6 > -15\)

These properties are also true for \(\leq\) and \(\geq\).

These properties can be used to solve inequalities. The solution sets of inequalities in one variable can then be graphed on number lines. Use a circle with an arrow to the left for \(<\) and an arrow to the right for \(>\). Use a dot with an arrow to the left for \(\leq\) and an arrow to the right for \(\geq\).
Example 1 Solve an Inequality Using Addition or Subtraction

Solve $7x - 5 > 6x + 4$. Graph the solution set on a number line.

Original inequality

Add $-6x$ to each side.

Add 5 to each side.

Any real number greater than 9 is a solution of this inequality.

The graph of the solution set is shown at the right.

CHECK Substitute 9 for $x$ in $7x - 5 > 6x + 4$. The two sides should be equal. Then substitute a number greater than 9. The inequality should be true.

Multiplying or dividing each side of an inequality by a positive number does not change the truth of the inequality. However, multiplying or dividing each side of an inequality by a negative number requires that the order of the inequality be reversed. For example, to reverse $\leq$, replace it with $\geq$.

### Key Concept

#### Properties of Inequality

**Multiplication Property of Inequality**

<table>
<thead>
<tr>
<th>Words</th>
<th>For any real numbers, $a$, $b$, and $c$, where...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$ is positive:</td>
<td>if $a &gt; b$, then $ac &gt; bc$.</td>
</tr>
<tr>
<td>$c$ is negative:</td>
<td>if $a &lt; b$, then $ac &lt; bc$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
<th>$-2 &lt; 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$4(-2) &lt; 4(3)$</td>
</tr>
<tr>
<td></td>
<td>$-8 &lt; 12$</td>
</tr>
<tr>
<td></td>
<td>$5 &gt; -1$</td>
</tr>
<tr>
<td></td>
<td>$(-3)(5) &lt; (-3)(1)$</td>
</tr>
<tr>
<td></td>
<td>$-15 &lt; 3$</td>
</tr>
</tbody>
</table>

**Division Property of Inequality**

<table>
<thead>
<tr>
<th>Words</th>
<th>For any real numbers, $a$, $b$, and $c$, where...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$ is positive:</td>
<td>if $a &gt; b$, then $\frac{a}{c} &gt; \frac{b}{c}$.</td>
</tr>
<tr>
<td>$c$ is negative:</td>
<td>if $a &lt; b$, then $\frac{a}{c} &lt; \frac{b}{c}$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
<th>$-18 &lt; -9$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-18 &lt; -9$</td>
</tr>
<tr>
<td></td>
<td>$\frac{3}{-3}$</td>
</tr>
<tr>
<td></td>
<td>$-6 &lt; -3$</td>
</tr>
<tr>
<td></td>
<td>$12 &gt; 8$</td>
</tr>
<tr>
<td></td>
<td>$\frac{12}{-2} &lt; \frac{8}{-2}$</td>
</tr>
<tr>
<td></td>
<td>$-6 &lt; -4$</td>
</tr>
</tbody>
</table>

These properties are also true for $\leq$ and $\geq$.

The solution set of an inequality can be expressed by using **set-builder notation**. For example, the solution set in Example 1 can be expressed as $\{x | x > 9\}$. 

---

Reading Math

{x | x > 9} is read the set of all numbers $x$ such that $x$ is greater than 9.
**Example 2** Solve an Inequality Using Multiplication or Division

Solve \(-0.25y \geq 2\). Graph the solution set on a number line.

\[
\begin{align*}
-0.25y & \geq 2 & \text{Original inequality} \\
\frac{-0.25y}{-0.25} & \leq \frac{2}{-0.25} & \text{Divide each side by \(-0.25\), reversing the inequality symbol.} \\
y & \leq -8 & \text{Simplify.}
\end{align*}
\]

The solution set is \(\{y \mid y \leq -8\}\).

The graph of the solution set is shown below.

A dot means that this point is included in the solution set.

**Study Tip**

Reading Math

The symbol \(+\infty\) is read positive infinity, and the symbol \(-\infty\) is read negative infinity.

**Example 3** Solve a Multi-Step Inequality

Solve \(-m \leq \frac{m + 4}{9}\). Graph the solution set on a number line.

\[
\begin{align*}
-m & \leq \frac{m + 4}{9} & \text{Original inequality} \\
-9m & \leq m + 4 & \text{Multiply each side by 9.} \\
-10m & \leq 4 & \text{Add \(-m\) to each side.} \\
m & \geq \frac{-4}{-10} & \text{Divide each side by \(-10\), reversing the inequality symbol.} \\
m & \geq \frac{2}{5} & \text{Simplify.}
\end{align*}
\]

The solution set is \(\left[-\frac{2}{5}, +\infty\right)\) and is graphed below.

The solution set of an inequality can also be described by using **interval notation**. The infinity symbols \(+\infty\) and \(-\infty\) are used to indicate that a set is unbounded in the positive or negative direction, respectively. To indicate that an endpoint is not included in the set, a parenthesis, ( or ), is used.

\[
\begin{align*}
x & < 2 & \text{interval notation} \\
(-\infty, 2)
\end{align*}
\]

A bracket is used to indicate that the endpoint, \(-2\), is included in the solution set below. Parentheses are always used with the symbols \(+\infty\) and \(-\infty\), because they do not include endpoints.

\[
\begin{align*}
x & \geq -2 & \text{interval notation} \\
[-2, +\infty)
\end{align*}
\]

www.algebra2.com/extra_examples
REAL-WORLD PROBLEMS WITH INEQUALITIES  Inequalities can be used to solve many verbal and real-world problems.

**Example 4 Write an Inequality**

DELIVERIES  Craig is delivering boxes of paper to each floor of an office building. Each box weighs 64 pounds, and Craig weighs 160 pounds. If the maximum capacity of the elevator is 2000 pounds, how many boxes can Craig safely take on each elevator trip?

Explore  Let \( b \) = the number of boxes Craig can safely take on each trip. A maximum capacity of 2000 pounds means that this weight must be less than or equal to 2000.

Plan  The total weight of the boxes is \( 64b \). Craig’s weight plus the total weight of the boxes must be less than or equal to 2000. Write an inequality.

\[
\begin{align*}
\text{Craig’s weight} & \quad \text{plus} \quad \text{the weight of the boxes} \quad \text{is less than or equal to} \quad 2000. \\
160 & \quad + \quad 64b & \quad \leq & \quad 2000
\end{align*}
\]

Solve  \[160 + 64b \leq 2000\]  Original inequality

\[
\begin{align*}
160 - 160 & \quad + \quad 64b \leq 2000 - 160 \\
64b & \quad \leq \quad 1840 \quad \text{Subtract 160 from each side.}
\end{align*}
\]

\[
\begin{align*}
64b & \quad \leq \quad 1840 \quad \text{Simplify.}
\end{align*}
\]

\[
\begin{align*}
\frac{64b}{64} & \quad \leq \quad \frac{1840}{64} \\
\frac{b}{1} & \quad \leq \quad \frac{1840}{64} \quad \text{Divide each side by 64.}
\end{align*}
\]

\[
\begin{align*}
b & \quad \leq \quad 28.75 \quad \text{Simplify.}
\end{align*}
\]

Examine  Since he cannot take a fraction of a box, Craig can take no more than 28 boxes per trip and still meet the safety requirements of the elevator.

You can use a graphing calculator to find the solution set for an inequality.

**Graphing Calculator Investigation**

**Solving Inequalities**

The inequality symbols in the TEST menu on the TI-83 Plus are called relational operators. They compare values and return 1 if the test is true or 0 if the test is false.

You can use these relational operators to find the solution set of an inequality in one variable.

**Think and Discuss**

1. Clear the \( Y= \) list. Enter \( 11x + 3 \geq 2x - 6 \) as \( Y_1 \). Put your calculator in DOT mode. Then, graph in the standard viewing window. Describe the graph.

2. Using the TRACE function, investigate the graph. What values of \( x \) are on the graph? What values of \( y \) are on the graph?

3. Based on your investigation, what inequality is graphed?

4. Solve \( 11x + 3 \geq 2x - 6 \) algebraically. How does your solution compare to the inequality you wrote in Exercise 3?
Check for Understanding

Concept Check

1. Explain why it is not necessary to state a division property for inequalities.
2. Write an inequality using the > symbol whose solution set is graphed below.

Guided Practice

Solve each inequality. Describe the solution set using set-builder or interval notation. Then graph the solution set on a number line.

4. \( a + 2 < 3.5 \)
5. \( 5 \geq 3x \)
6. \( 11 - c \leq 8 \)
7. \( 4y + 7 > 31 \)
8. \( 2w + 19 < 5 \)
9. \( -0.6p < -9 \)
10. \( \frac{n}{12} + 15 \leq 13 \)
11. \( \frac{5x + 2}{4} < \frac{5x}{4} + 2 \)

Define a variable and write an inequality for each problem. Then solve.

12. The product of 12 and a number is greater than 36.
13. Three less than twice a number is at most 5.

Application

14. SCHOOL  The final grade for a class is calculated by taking 75% of the average test score and adding 25% of the score on the final exam. If all scores are out of 100 and a student has a 76 test average, what score does the student need to make on the final exam to have a final grade of at least 80?

Practice and Apply

Solve each inequality. Describe the solution set using set-builder or interval notation. Then, graph the solution set on a number line.

15. \( n + 4 \geq -7 \)
16. \( b - 3 \leq 15 \)
17. \( 5x < 35 \)
18. \( \frac{d}{2} > -4 \)
19. \( \frac{8}{3} \geq -9 \)
20. \( -8p \geq 24 \)
21. \( 13 - 4k \leq 27 \)
22. \( 14 > 7y - 21 \)
23. \( -27 < 8m + 5 \)
24. \( 6b + 11 \geq 15 \)
25. \( 2(4t + 9) \leq 18 \)
26. \( 90 \geq 5(2r + 6) \)
27. \( 14 - 8n \leq 0 \)
28. \( -4(5w - 8) < 33 \)
29. \( 0.02x + 5.58 < 0 \)
30. \( 1.5 - 0.25c < 6 \)
31. \( 6d + 3 \geq 5d - 2 \)
32. \( 9z + 2 > 4z + 15 \)
33. \( 2(g + 4) < 3g - 2(g - 5) \)
34. \( 3(a + 4) - 2(3a + 4) \leq 4a - 1 \)
35. \( y < -\frac{y + 2}{9} \)
36. \( \frac{1 - 4p}{5} < 0.2 \)
37. \( \frac{4x + 2}{6} < \frac{2x + 1}{3} \)
38. \( 12\left(\frac{1}{4} - \frac{n}{3}\right) \leq -6n \)

39. PART-TIME JOB  David earns $5.60 an hour working at Box Office Videos. Each week, 25% of his total pay is deducted for taxes. If David wants his take-home pay to be at least $105 a week, solve the inequality \( 5.6x - 0.25(5.6x) \geq 105 \) to determine how many hours he must work.

40. STATE FAIR  Juan’s parents gave him $35 to spend at the State Fair. He spends $13.25 for food. If rides at the fair cost $1.50 each, solve the inequality \( 1.5n + 13.25 \leq 35 \) to determine how many rides he can afford.
Define a variable and write an inequality for each problem. Then solve.

41. The sum of a number and 8 is more than 2.
42. The product of $-4$ and a number is at least 35.
43. The difference of one half of a number and 7 is greater than or equal to 5.
44. One more than the product of $-3$ and a number is less than 16.
45. Twice the sum of a number and 5 is no more than 3 times that same number increased by 11.
46. 9 less than a number is at most that same number divided by 2.

47. **CHILD CARE**  By Ohio law, when children are napping, the number of children per child care staff member may be as many as twice the maximum listed at the right. Write and solve an inequality to determine how many staff members are required to be present in a room where 17 children are napping and the youngest child is 18 months old.

<table>
<thead>
<tr>
<th>Maximum Number of Children Per Child Care Staff Member</th>
</tr>
</thead>
<tbody>
<tr>
<td>At least one child care staff member caring for:</td>
</tr>
<tr>
<td>Every 5 infants less than 12 months old (or 2 for every 12)</td>
</tr>
<tr>
<td>Every 6 infants who are at least 12 months old, but less than 18 months old</td>
</tr>
<tr>
<td>Every 7 toddlers who are at least 18 months old, but less than 30 months old</td>
</tr>
<tr>
<td>Every 8 toddlers who are at least 30 months old, but less than 3 years old</td>
</tr>
</tbody>
</table>

Source: Ohio Department of Job and Family Services

**CAR SALES**  For Exercises 48 and 49, use the following information.
Mrs. Lucas earns a salary of $24,000 per year plus 1.5% commission on her sales. If the average price of a car she sells is $30,500, about how many cars must she sell to make an annual income of at least $40,000?

48. Write an inequality to describe this situation.
49. Solve the inequality and interpret the solution.

**TEST GRADES**  For Exercises 50 and 51, use the following information.
Ahmik’s scores on the first four of five 100-point history tests were 85, 91, 89, and 94.

50. If a grade of at least 90 is an A, write an inequality to find the score Ahmik must receive on the fifth test to have an A test average.
51. Solve the inequality and interpret the solution.

52. **CRITICAL THINKING**  Which of the following properties hold for inequalities?

   a. Reflexive  
   b. Symmetric  
   c. Transitive

53. **WRITING IN MATH**  Answer the question that was posed at the beginning of the lesson.

   **How can inequalities be used to compare phone plans?**

   Include the following in your answer:

   • an inequality comparing the number of minutes offered by each plan, and
   • an explanation of how Kuni might determine when Plan 1 might be cheaper than Plan 2 if she typically uses more than 150 but less than 400 minutes.
Standardized Test Practice

54. If \(4 - 5n \geq -1\), then \(n\) could equal all of the following EXCEPT

- \(A\) \(-1/5\)
- \(B\) \(1/5\)
- \(C\) 1.
- \(D\) 2.

55. If \(a < b\) and \(c < 0\), which of the following are true?

- \(I\). \(ac > bc\)
- \(II\). \(a + c < b + c\)
- \(III\). \(a - c > b - c\)

- \(A\) I only
- \(B\) II only
- \(C\) III only
- \(D\) I, II, and III

Graphing Calculator

Use a graphing calculator to solve each inequality.

56. \(-5x - 8 < 7\)
57. \(-4(6x - 3) \leq 60\)
58. \(3(x + 3) \leq 2(x + 4)\)

Maintain Your Skills

Mixed Review

Solve each equation. Check your solutions. (Lesson 1-4)

59. \(|x - 3| = 17\)
60. \(8|4x - 3| = 64\)
61. \(|x + 1| = x\)

62. SHOPPING On average, by how much did the number of people who just browse, but not necessarily buy, online increase each year from 1997 to 2003? Define a variable, write an equation, and solve the problem. (Lesson 1-3)

Name the sets of numbers to which each number belongs. (Lesson 1-2)

63. 31
64. \(-4.2\)
65. \(\sqrt{7}\)

66. BABY-SITTING Jenny baby-sat for 5 \(\frac{1}{2}\) hours on Friday night and 8 hours on Saturday. She charges $4.25 per hour. Use the Distributive Property to write two equivalent expressions that represent how much money Jenny earned. (Lesson 1-2)

Getting Ready for the Next Lesson

PREREQUISITE SKILL Solve each equation. Check your solutions. (To review solving absolute value equations, see Lesson 1-4.)

67. \(|x| = 7\)
68. \(|x + 5| = 18\)
69. \(|5y - 8| = 12\)
70. \(|2x - 36| = 14\)
71. \(2|w + 6| = 10\)
72. \(|x + 4| + 3 = 17\)

Practice Quiz 2

Lessons 1-3 through 1-5

1. Solve \(2d + 5 = 8d + 2\). Check your solution. (Lesson 1-3)
2. Solve \(s = \frac{1}{2}gt^2\) for \(g\). (Lesson 1-3)
3. Evaluate \(|x - 3y|\) if \(x = -8\) and \(y = 2\). (Lesson 1-4)
4. Solve \(3|3x + 2| = 51\). Check your solutions. (Lesson 1-4)
5. Solve \(2(m - 5) - 3(2m - 5) < 5m + 1\). Describe the solution set using set-builder or interval notation. Then graph the solution set on a number line. (Lesson 1-5)
Solving Compound and Absolute Value Inequalities

**What You’ll Learn**

- Solve compound inequalities.
- Solve absolute value inequalities.

**Vocabulary**

- compound inequality
- intersection
- union

**How are compound inequalities used in medicine?**

One test used to determine whether a patient is diabetic and requires insulin is a glucose tolerance test. Patients start the test in a fasting state, meaning they have had no food or drink except water for at least 10 but no more than 16 hours. The acceptable number of hours $h$ for fasting can be described by the following compound inequality.

$$h \geq 10 \text{ and } h \leq 16$$

**COMPOUND INEQUALITIES**

A compound inequality consists of two inequalities joined by the word and or the word or. To solve a compound inequality, you must solve each part of the inequality. The graph of a compound inequality containing and is the intersection of the solution sets of the two inequalities.

**Key Concept**

**“And” Compound Inequalities**

- **Words**
  
  A compound inequality containing the word and is true if and only if both inequalities are true.

- **Example**

  $x \geq -1$

  $\quad$ $x < 2$

  $x \geq -1 \text{ and } x < 2$

  Another way of writing $x \geq -1$ and $x < 2$ is $-1 \leq x < 2$.

  Both forms are read $x$ is greater than or equal to $-1$ and less than 2.

**Example 1**

**Solve an “and” Compound Inequality**

Solve $13 < 2x + 7 \leq 17$. Graph the solution set on a number line.

**Method 1**

Write the compound inequality using the word and. Then solve each inequality.

- $13 < 2x + 7$
  - $6 < 2x$
  - $3 < x$

- $2x + 7 \leq 17$
  - $2x \leq 10$
  - $x \leq 5$

**Method 2**

Solve both parts at the same time by subtracting 7 from each part. Then divide each part by 2.

- $13 < 2x + 7 \leq 17$
  - $6 < 2x \leq 10$
  - $3 < x \leq 5$
Graph the solution set for each inequality and find their intersection.

\[ \begin{align*}
  &0, 1, 2, 3, 4, 5, 6, 7, 8 \\
\end{align*} \]

- For \( x > 3 \):
  \[ \begin{align*}
  &-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8 \\
\end{align*} \]
- For \( x \leq 5 \):
  \[ \begin{align*}
  &-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8 \\
\end{align*} \]
- For \( 3 < x \leq 5 \):
  \[ \begin{align*}
  &-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8 \\
\end{align*} \]

The solution set is \( \{ x \mid 3 < x \leq 5 \} \).

The graph of a compound inequality containing or is the union of the solution sets of the two inequalities.

**Key Concept**

**“Or” Compound Inequalities**

- **Words**: A compound inequality containing the word or is true if one or more of the inequalities is true.
- **Example**

  \[ \begin{align*}
  x &\leq 1 \\
  &-2, -1, 0, 1, 2, 3, 4, 5, 6 \\
  x &> 4 \\
  &-2, -1, 0, 1, 2, 3, 4, 5, 6 \\
  x &\leq 1 \text{ or } x > 4 \\
  &-2, -1, 0, 1, 2, 3, 4, 5, 6 \\
\end{align*} \]

**Example 2**

**Solve an “or” Compound Inequality**

Solve \( y - 2 > -3 \) or \( y + 4 \leq -3 \). Graph the solution set on a number line.

Solve each inequality separately.

- \( y - 2 > -3 \) \quad or \quad \( y + 4 \leq -3 \)
- \( y > -1 \) \quad or \quad \( y \leq -7 \)

The solution set is \( \{ y \mid y > -1 \text{ or } y \leq -7 \} \).

**ABSOLUTE VALUE INEQUALITIES**

In Lesson 1-4, you learned that the absolute value of a number is its distance from 0 on the number line. You can use this definition to solve inequalities involving absolute value.
Example 3 Solve an Absolute Value Inequality (<)

Solve \(|a| < 4\). Graph the solution set on a number line.

You can interpret \(|a| < 4\) to mean that the distance between \(a\) and 0 on a number line is less than 4 units. To make \(|a| < 4\) true, you must substitute numbers for \(a\) that are fewer than 4 units from 0.

All of the numbers between \(-4\) and 4 are less than 4 units from 0. The solution set is \(\{a \mid -4 < a < 4\}\).

Example 4 Solve an Absolute Value Inequality (>)

Solve \(|a| > 4\). Graph the solution set on a number line.

You can interpret \(|a| > 4\) to mean that the distance between \(a\) and 0 is greater than 4 units. To make \(|a| > 4\) true, you must substitute values for \(a\) that are greater than 4 units from 0.

All of the numbers not between \(-4\) and 4 are greater than 4 units from 0. The solution set is \(\{a \mid a > 4\} \text{ or } \{a \mid a < -4\}\).

An absolute value inequality can be solved by rewriting it as a compound inequality.

Example 5 Solve a Multi-Step Absolute Value Inequality

Solve \(|3x - 12| \geq 6\). Graph the solution set on a number line.

\(|3x - 12| \geq 6\) is equivalent to \(3x - 12 \geq 6\) or \(3x - 12 \leq -6\). Solve each inequality.

\[3x - 12 \geq 6\]  \[3x \geq 18\]  \[x \geq 6\]
\[3x - 12 \leq -6\]  \[3x \leq 6\]  \[x \leq 2\]

The solution set is \(\{x \mid x \geq 6 \text{ or } x \leq 2\}\).
Lesson 1-6

Solving Compound and Absolute Value Inequalities

Example 6

Write an Absolute Value Inequality

JOB HUNTING

To prepare for a job interview, Megan researches the position’s requirements and pay. She discovers that the average starting salary for the position is $38,500, but her actual starting salary could differ from the average by as much as $2450.

a. Write an absolute value inequality to describe this situation.

Let \( x \) be Megan’s starting salary.

Her starting salary could differ from the average by as much as $2450.

\[
|38,500 - x| \leq 2450
\]

b. Solve the inequality to find the range of Megan’s starting salary.

Rewrite the absolute value inequality as a compound inequality.

Then solve for \( x \).

\[
-2450 \leq 38,500 - x \leq 2450
\]

\[
38,500 - 2450 \leq 38,500 - x \leq 38,500 + 2450
\]

\[
-40,950 \leq -x \leq 63,050
\]

\[
x \geq 63,050
\]

The solution set is \( \{x | 36,050 \leq x \leq 40,950\} \). Thus, Megan’s starting salary will fall between $36,050 and $40,950, inclusive.

Check for Understanding

Concept Check

1. Write a compound inequality to describe the following situation.

Buy a present that costs at least $5 and at most $15.

2. OPEN ENDED

Write a compound inequality whose graph is the empty set.

3. FIND THE ERROR

Sabrina and Isaac are solving \( |3x + 7| > 2 \).

Sabrina

\[
3x + 7 > 2 \quad \text{or} \quad 3x + 7 < -2
\]

\[
x > -\frac{5}{3}
\]

Isaac

\[
3x + 7 > 2
\]

\[
-2 < 3x + 7 < 2
\]

\[
x > -\frac{5}{3}
\]

Who is correct? Explain your reasoning.

Guided Practice

Write an absolute value inequality for each of the following. Then graph the solution set on a number line.

4. all numbers between \(-8\) and \(8\)

5. all numbers greater than \(3\) and less than \(-3\)

Write an absolute value inequality for each graph.

6.

7.
Betta Fish

**Adult Male Size:** 3 inches

**Water pH:** 6.8–7.4

**Temperature:** 75–86°F

**Diet:** omnivore, prefers live foods

**Tank Level:** top dweller

**Difficulty of Care:** easy to intermediate

**Life Span:** 2–3 years

**Source:** www.about.com

Solve each inequality. Graph the solution set on a number line.

8. \( y - 3 > 1 \) or \( y + 2 < 1 \)

9. \( 3 < d + 5 < 8 \)

10. \( |a| \geq 5 \)

11. \( |g + 4| \leq 9 \)

12. \( |4k - 8| < 20 \)

13. \( |w| \geq -2 \)

**Application**

**FLOORING** Deion estimates that he will need between 55 and 60 ceramic tiles to retile his kitchen floor. If each tile costs $6.25, write and solve a compound inequality to determine what the cost \( c \) of the tile could be.

**Practice and Apply**

Write an absolute value inequality for each of the following. Then graph the solution set on a number line.

15. all numbers greater than or equal to 5 or less than or equal to 2

16. all numbers less than 7 and greater than 7

17. all numbers between 4 and 4

18. all numbers less than or equal to 6 or greater than or equal to 6

19. all numbers greater than 8 or less than 8

20. all number less than or equal to 1.2 and greater than or equal to 1.2

Write an absolute value inequality for each graph.

21.

22.

23.

24.

25.

26.

Solve each inequality. Graph the solution set on a number line.

27. \( 3p + 1 \leq 7 \) or \( 2p - 9 \geq 7 \)

28. \( 9 < 3t + 6 < 15 \)

29. \( -11 < -4x + 5 < 13 \)

30. \( 2c - 1 < -5 \) or \( 3c + 2 \geq 5 \)

31. \( -4 < 4f + 24 < 4 \)

32. \( a + 2 > -2 \) or \( a - 8 < 1 \)

33. \( |g| \leq 9 \)

34. \( |2m| \geq 8 \)

35. \( |3k| > 0 \)

36. \( |-5y| < 35 \)

37. \( |b - 4| > 6 \)

38. \( |6r - 3| < 21 \)

39. \( |3w + 2| \leq 5 \)

40. \( |7x| + 4 < 0 \)

41. \( |n| \geq n \)

42. \( |n| \leq n \)

43. \( |2n - 7| \leq 0 \)

44. \( |n - 3| < n \)

**Homework Help**

**Extra Practice**

See page 829.

45. **BETTA FISH** A Siamese Fighting Fish, also known as a Betta fish, is one of the most recognized and colorful fish kept as a pet. Using the information at the left, write a compound inequality to describe the acceptable range of water pH levels for a male Betta.
SPEED LIMITS For Exercises 46 and 47, use the following information.
On some interstate highways, the maximum speed a car may drive is 65 miles per hour. A tractor-trailer may not drive more than 55 miles per hour. The minimum speed for all vehicles is 45 miles per hour.

46. Write an inequality to represent the allowable speed for a car on an interstate highway.
47. Write an inequality to represent the speed at which a tractor-trailer may travel on an interstate highway.

48. HEALTH Hypothermia and hyperthermia are similar words but have opposite meanings. Hypothermia is defined as a lowered body temperature. Hyperthermia means an extremely high body temperature. Both conditions are potentially dangerous and occur when a person’s body temperature fluctuates by more than 8° from the normal body temperature of 98.6°F. Write and solve an absolute value inequality to describe body temperatures that are considered potentially dangerous.

MAIL For Exercises 49 and 50, use the following information.
The U.S. Postal Service defines an oversized package as one for which the length $L$ of its longest side plus the distance $D$ around its thickest part is more than 108 inches and less than or equal to 130 inches.

49. Write a compound inequality to describe this situation.
50. If the distance around the thickest part of a package you want to mail is 24 inches, describe the range of lengths that would classify your package as oversized.

GEOMETRY For Exercises 51 and 52, use the following information.
The Triangle Inequality Theorem states that the sum of the measures of any two sides of a triangle is greater than the measure of the third side.

51. Write three inequalities to express the relationships among the sides of $\triangle ABC$.
52. Write a compound inequality to describe the range of possible measures for side $c$ in terms of $a$ and $b$. Assume that $a > b > c$. (Hint: Solve each inequality you wrote in Exercise 51 for $c$.)

53. CRITICAL THINKING Graph each set on a number line.
   a. $-2 < x < 4$
   b. $x < -1$ or $x > 3$
   c. $(-2 < x < 4)$ and $(x < -1$ or $x > 3)$ (Hint: This is the intersection of the graphs in part a and part b.)
   d. Solve $3 < |x + 2| \leq 8$. Explain your reasoning and graph the solution set.

54. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.
How are compound inequalities used in medicine?
Include the following in your answer:
• an explanation as to when to use and and when to use or when writing a compound inequality,
• an alternative way to write $h \geq 10$ and $h \leq 16$, and
• an example of an acceptable number of hours for this fasting state and a graph to support your answer.
55. SHORT RESPONSE  Solve $|2x + 11| > 1$ for $x$.

56. If $5 < a < 7 < b < 14$, then which of the following best defines $\frac{a}{b}$?

A. $\frac{5}{7} < \frac{a}{b} < \frac{7}{14}$  
B. $\frac{5}{7} < \frac{a}{b} < \frac{1}{2}$  
C. $\frac{5}{7} < \frac{a}{b} < 1$  
D. $\frac{5}{14} < \frac{a}{b} < \frac{7}{14}$

LOGIC MENU  For Exercises 57–60, use the following information.
You can use the operators in the LOGIC menu on the TI-83 Plus to graph compound and absolute value inequalities. To display the LOGIC menu, press \text{2nd} \text{TEST} \text{DISTR}.

57. Clear the Y= list. Enter $(5x + 2 > 12)$ and $(3x - 8 < 1)$ as Y1. With your calculator in DOT mode and using the standard viewing window, press \text{GRAPH}. Make a sketch of the graph displayed.

58. Using the TRACE function, investigate the graph. Based on your investigation, what inequality is graphed?

59. Write the expression you would enter for Y1 to find the solution set of the compound inequality $5x + 2 \geq 3$ or $5x + 2 \leq -3$. Then use the graphing calculator to find the solution set.

60. A graphing calculator can also be used to solve absolute value inequalities. Write the expression you would enter for Y1 to find the solution set of the inequality $|2x - 6| > 10$. Then use the graphing calculator to find the solution set. (Hint: The absolute value operator is item 1 on the MATH NUM menu.)

Maintain Your Skills

Solve each inequality. Describe the solution set using set builder or interval notation. Then graph the solution set on a number line. \text{(Lesson 1-5)}

61. $2d + 15 \geq 3$  
62. $7x + 11 > 9x + 3$  
63. $3n + 4(n + 3) < 5(n + 2)$

64. CONTESTS  To get a chance to win a car, you must guess the number of keys in a jar to within 5 of the actual number. Those who are within this range are given a key to try in the ignition of the car. Suppose there are 587 keys in the jar. Write and solve an equation to determine the highest and lowest guesses that will give contestants a chance to win the car. \text{(Lesson 1-4)}

Solve each equation. Check your solutions.

65. $5| x - 3| = 65$  
66. $|2x + 7| = 15$  
67. $|8c + 7| = -4$

Name the property illustrated by each statement. \text{(Lesson 1-3)}

68. If $3x = 10$, then $3x + 7 = 10 + 7$.
69. If $-5 = 4y - 8$, then $4y - 8 = -5$.
70. If $-2x - 5 = 9$ and $9 = 6x + 1$, then $-2x - 5 = 6x + 1$.

Simplify each expression. \text{(Lesson 1-2)}

71. $6a - 2b - 3a + 9b$  
72. $-2(m - 4n) - 3(5n + 6)$

Find the value of each expression. \text{(Lesson 1-1)}

73. $6(5 - 8) + 9 + 4$  
74. $(3 + 7)^2 - 16 ÷ 2$  
75. $\frac{7(1 - 4)}{8 - 5}$
Choose the term from the list above that best matches each example.

1. \( y > 3 \) or \( y < -2 \)

2. \( 0 + (-4b) = -4b \)

3. \((m - 1)(-2) = -2(m - 1)\)

4. \( 35x + 56 = 7(5x + 8) \)

5. \( ab + 1 = ab + 1 \)

6. If \( 2x = 3y - 4, \ 3y - 4 = 7 \), then \( 2x = 7 \).

7. \( 4(0.25) = 1 \)

8. \( 2p + (4 + 9r) = (2p + 4) + 9r \)

9. \( |5n| \)

10. \( 6y + 5z - 2(x + y) \)

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**Lesson-by-Lesson Review**

**1-1 Expressions and Formulas**

**Concept Summary**

- Order of Operations

  **Step 1** Simplify the expressions inside grouping symbols, such as parentheses, ( ), brackets, [ ], braces, { }, and fraction bars.

  **Step 2** Evaluate all powers.

  **Step 3** Do all multiplications and/or divisions from left to right.

  **Step 4** Do all additions and/or subtractions from left to right.

**Example**

Evaluate \( \frac{y^3}{3ab + 2} \) if \( y = 4, \ a = -2, \) and \( b = -5 \).

\[
\frac{y^3}{3ab + 2} = \frac{4^3}{3(-2)(-5) + 2} = \frac{64}{3(10) + 2} = \frac{64}{32} = 2
\]

Evaluate the numerator and denominator separately.
Exercises  Find the value of each expression.  See Example 1 on page 6.

11.  $10 + 16 \div 4 + 8$  
12.  $[21 - (9 - 2)] \div 2$  
13.  $\frac{14(8 - 15)}{2}$

Evaluate each expression if  $a = 12$,  $b = 0.5$,  $c = -3$, and  $d = \frac{1}{3}$.

See Examples 2 and 3 on page 7.

14.  $6b - 5c$  
15.  $c^3 + ad$  
16.  $\frac{9c + ab}{c}$  
17.  $a[b^2(b + a)]$

1-2  Properties of Real Numbers

Concept Summary
- Real numbers (R) can be classified as rational (Q) or irrational (I).
- Rational numbers can be classified as natural numbers (N), whole numbers (W), and/or integers (Z).
- Use the properties of real numbers to simplify algebraic expressions.

Example
Simplify $4(2b + 6c) + 3b - c$.

$4(2b + 6c) + 3b - c = 4(2b) + 4(6c) + 3b - c$  
Distributive Property

$= 8b + 24c + 3b - c$  
Multiply.

$= 8b + 3b + 24c - c$  
Commutative Property (+)

$= (8 + 3)b + (24 - 1)c$  
Distributive Property

$= 11b + 23c$  
Add 3 to 8 and subtract 1 from 24.

Exercises  Name the sets of numbers to which each value belongs.  See Example 1 on page 12.

18.  $-\sqrt{9}$  
19.  $1.6$  
20.  $\frac{35}{7}$  
21.  $\sqrt{18}$

Simplify each expression.  See Example 5 on page 14.

22.  $2m + 7n - 6m - 5n$  
23.  $-5(a - 4b) + 4b$  
24.  $2(5x + 4y) - 3(x + 8y)$

1-3  Solving Equations

Concept Summary
- Verbal expressions can be translated into algebraic expressions using the language of algebra, using variables to represent the unknown quantities.
- Use the properties of equality to solve equations.

Example
Solve $4(a + 5) - 2(a + 6) = 3$.

$4(a + 5) - 2(a + 6) = 3$  
Original equation

$4a + 20 - 2a - 12 = 3$  
Distributive Property

$2a + 8 = 3$  
Commutative, Distributive, and Substitution Properties

$2a = -5$  
Subtraction Property (=)

$a = -2.5$  
Division Property (=)
Chapter 1  Study Guide and Review

**Exercises**  Solve each equation. Check your solution.  
See Examples 3 and 4 on pages 21 and 22.

25. \( x - 6 = -20 \)  
26. \( -\frac{2}{3}a = 14 \)  
28. \( 3w + 14 = 7w + 2 \)  
29. \( 5y + 4 = 2(y - 4) \)  
30. \( \frac{n}{4} + \frac{n}{3} = \frac{1}{2} \)

**Solve each equation or formula for the specified variable.**  See Example 5 on page 22.

31. \( Ax + By = C \) for \( x \) 
32. \( \frac{a - 4b^2}{2c} = d \) for \( a \) 
33. \( A = p + prt \) for \( p \)

---

### Solving Absolute Value Equations

**Concept Summary**
- For any real numbers \( a \) and \( b \), where \( b \neq 0 \), if \( |a| = b \), then \( a = b \) or \( a = -b \).

**Example**
Solve \( |2x + 9| = 11 \).

Case 1  \( a = b \)  
Case 2  \( a = -b \)

\[
\begin{align*}
2x + 9 &= 11 \\
2x &= 2 \\
x &= 1 \\
\end{align*}
\]

\[
\begin{align*}
2x + 9 &= -11 \\
2x &= -20 \\
x &= -10 \\
\end{align*}
\]

The solution set is \( \{1, -10\} \). Check these solutions in the original equation.

**Exercises**  Solve each equation. Check your solutions.  
See Examples 1–4 on pages 28–30.

34. \( |x + 11| = 42 \)  
35. \( 3|x + 6| = 36 \)  
36. \( |4x - 5| = -25 \)  
37. \( |x + 7| = 3x - 5 \)  
38. \( |y - 5| - 2 = 10 \)  
39. \( 4|3x + 4| = 4x + 8 \)

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### Solving Inequalities

**Concept Summary**
- Adding the same number to, or subtracting the same number from, each side of an inequality does not change the truth of the inequality.
- When you multiply or divide each side of an inequality by a negative number, the direction of the inequality symbol must be reversed.

**Example**
Solve \( 5 - 4a > 8 \). Graph the solution set on a number line.

\[
\begin{align*}
5 - 4a &> 8 \quad \text{Original inequality} \\
-4a &> 3 \quad \text{Subtract 5 from each side.} \\
a &< -\frac{3}{4} \quad \text{Divide each side by -4, reversing the inequality symbol.} \\
\end{align*}
\]

The solution set is \( \left[a \bigg| a < -\frac{3}{4}\right] \).

The graph of the solution set is shown at the right.
Exercises  Solve each inequality. Describe the solution set using set builder or interval notation. Then graph the solution set on a number line. See Examples 1–3 on pages 34–35.

40. \(-7w > 28\)  
41. \(3x + 4 \geq 19\)  
42. \(\frac{n}{12} + 5 \leq 7\)  
43. \(3(6 - 5a) < 12a - 36\)  
44. \(2 - 3z \geq 7(8 - 2z) + 12\)  
45. \(8(2x - 1) > 11x - 17\)

Solving Compound and Absolute Value Inequalities

Concept Summary

- The graph of an and compound inequality is the intersection of the solution sets of the two inequalities.
- The graph of an or compound inequality is the union of the solution sets of the two inequalities.
- For all real numbers \(a\) and \(b\), \(b > 0\), the following statements are true.
  1. If \(|a| < b\) then \(-b < a < b\).
  2. If \(|a| > b\) then \(a > b\) or \(a < -b\).

Examples  Solve each inequality. Graph the solution set on a number line.

1. \(-19 < 4d - 7 \leq 13\)

   \(-19 < 4d - 7 \leq 13\) Original inequality

   \(-12 < 4d \leq 20\) Add 7 to each part.

   \(-3 < d \leq 5\) Divide each part by 4.

   The solution set is \(\{x \mid -3 < d \leq 5\}\).

2. \(|2x + 4| \geq 12\)

   \(|2x + 4| \geq 12\) is equivalent to \(2x + 4 \geq 12\) or \(2x + 4 \leq -12\).

   \(2x + 4 \geq 12\) or \(2x + 4 \leq -12\) Original inequality

   \(2x \geq 8\) or \(2x \leq -16\) Subtract 4 from each side.

   \(x \geq 4\) or \(x \leq -8\) Divide each side by 2.

   The solution set is \(\{x \mid x \geq 4\) or \(x \leq -8\}\).

Exercises  Solve each inequality. Graph the solution set on a number line. See Examples 1–5 on pages 40–42.

46. \(-1 < 3a + 2 < 14\)  
47. \(-1 < 3(y - 2) \leq 9\)  
48. \(|x| + 1 > 12\)  
49. \(|2y - 9| \leq 27\)  
50. \(|5n - 8| > -4\)  
51. \(|3b + 11| > 1\)
10. **Name the sets of numbers to which each number belongs.**

Find the value of each expression.

4. \[(3 + 6)^2 ÷ 3 \times 4\]

5. \[\frac{20 + 4 \times 3}{11 - 3}\]

6. \[0.5(2.3 + 25) ÷ 1.5\]

Evaluate each expression if \(a = -9, b = \frac{2}{3}, c = 8, \) and \(d = -6.\)

7. \[ \frac{db + 4c}{a} \]

8. \[ \frac{a}{b^2} + c \]

9. \[2b(4a + a^2)\]

**Name the property illustrated by each equation or statement.**

13. \((7 \cdot s) \cdot t = 7 \cdot (s \cdot t)\)

14. If \((r + s)t = rt + st,\) then \(rt + st = (r + s)t.\)

15. \((3 \cdot \frac{1}{3}) \cdot 7 = (\frac{3}{1}) \cdot 7\)

16. \((6 - 2)a - 3b = 4a - 3b\)

17. \((4 + x) + y = y + (4 + x)\)

18. If \(5(3) + 7 = 15 + 7\) and \(15 + 7 = 22,\)
then \(5(3) + 7 = 22.\)

**Solve each equation. Check your solution(s).**

19. \(5t - 3 = -2t + 10\)

20. \(2x - 7 - (x - 5) = 0\)

21. \(5m - (5 + 4m) = (3 + m) - 8\)

22. \(|8w + 2| + 2 = 0\)

23. \(12\left|\frac{1}{2}y + 3\right| = 6\)

24. \(2\left|2y - 6\right| + 4 = 8\)

**Solve each inequality. Describe the solution set using set builder or interval notation.**

Then graph the solution set on a number line.

25. \(4 > b + 1\)

26. \(3q + 7 \geq 13\)

27. \(5(3x - 5) + x < 2(4x - 1) + 1\)

28. \(|5 + k| \leq 8\)

29. \(-12 < 7d - 5 \leq 9\)

30. \(|3y - 1| > 5\)

**For Exercises 31 and 32, define a variable, write an equation or inequality, and solve the problem.**

31. **CAR RENTAL** Mrs. Denney is renting a car that gets 35 miles per gallon. The rental charge is $19.50 a day plus 18¢ per mile. Her company will reimburse her for $33 of this portion of her travel expenses. If Mrs. Denney rents the car for 1 day, find the maximum number of miles that will be paid for by her company.

32. **SCHOOL** To receive a B in his English class, Nick must have an average score of at least 80 on five tests. He scored 87, 89, 76, and 77 on his first four tests. What must he score on the last test to receive a B in the class?

33. **STANDARDIZED TEST PRACTICE** If \(\frac{a}{b} = 8\) and \(ac - 5 = 11,\) then \(bc =\)

A. 93.

B. 2.

C. \(\frac{5}{8}\).

D. cannot be determined

www.algebra2.com/chapter_test
Part 1  Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. In the square at the right, what is the value of \( x + 8 \) ?
   \[ \begin{array}{c}
   \text{A} \ 1 \\
   \text{B} \ 2 \\
   \text{C} \ 3 \\
   \text{D} \ 4
   \end{array} \]

2. On a college math test, 18 students earned an A. This number is exactly 30% of the total number of students in the class. How many students are in the class?
   \[ \begin{array}{c}
   \text{A} \ 52 \\
   \text{B} \ 3 \\
   \text{C} \ 48 \\
   \text{D} \ 60
   \end{array} \]

3. A student computed the average of her 7 test scores by adding the scores together and dividing this total by the number of tests. The average was 87. On her next test, she scored a 79. What is her new test average?
   \[ \begin{array}{c}
   \text{A} \ 83 \\
   \text{B} \ 84 \\
   \text{C} \ 85 \\
   \text{D} \ 86
   \end{array} \]

4. If the perimeter of \( \triangle PQR \) is 3 times the length of \( PQ \), then \( PR = \) ?
   \[ \begin{array}{c}
   \text{A} \ 4 \\
   \text{B} \ 6 \\
   \text{C} \ 7 \\
   \text{D} \ 8
   \end{array} \]

5. If a different number is selected from each of the three sets shown below, what is the greatest sum these 3 numbers could have?
   \[ R = \{3, 6, 7\}; S = \{2, 4, 7\}; T = \{1, 3, 7\} \]
   \[ \begin{array}{c}
   \text{A} \ 13 \\
   \text{B} \ 14 \\
   \text{C} \ 17 \\
   \text{D} \ 21
   \end{array} \]

6. A pitcher contains \( a \) ounces of orange juice. If \( b \) ounces of juice are poured from the pitcher into each of \( c \) glasses, which expression represents the amount of juice remaining in the pitcher?
   \[ \begin{array}{c}
   \text{A} \ \frac{a}{b} + c \\
   \text{B} \ ab - c \\
   \text{C} \ a - bc \\
   \text{D} \ \frac{a}{bc}
   \end{array} \]

7. The sum of three consecutive integers is 135. What is the greatest of the three integers?
   \[ \begin{array}{c}
   \text{A} \ 43 \\
   \text{B} \ 44 \\
   \text{C} \ 45 \\
   \text{D} \ 46
   \end{array} \]

8. The ratio of girls to boys in a class is 5 to 4. If there are a total of 27 students in the class, how many are girls?
   \[ \begin{array}{c}
   \text{A} \ 15 \\
   \text{B} \ 12 \\
   \text{C} \ 9 \\
   \text{D} \ 5
   \end{array} \]

9. For which of the following ordered pairs \((x, y)\) is \( x + y > 3 \) and \( x - y < -2 \)?
   \[ \begin{array}{c}
   \text{A} \ (0, 3) \\
   \text{B} \ (3, 4) \\
   \text{C} \ (5, 3) \\
   \text{D} \ (2, 5)
   \end{array} \]

10. If the area of \( \triangle ABD \) is 280, what is the area of the polygon \( ABCD \)?
   \[ \begin{array}{c}
   \text{A} \ 560 \\
   \text{B} \ 630 \\
   \text{C} \ 700 \\
   \text{D} \ 840
   \end{array} \]

Test-Taking Tip

Question 9  To solve equations or inequalities, you can replace the variables in the question with the values given in each answer choice. The answer choice that results in true statements is the correct answer choice.
Part 2  Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. In the triangle below, \( x \) and \( y \) are integers. If \( 25 < y < 30 \), what is one possible value of \( x \)?

![Triangle diagram]

12. If \( n \) and \( p \) are each different positive integers and \( n + p = 4 \), what is one possible value of \( 3n + 4p \)?

13. In the figure at the right, what is the value of \( x \)?

![Angle diagram]

14. One half quart of lemonade concentrate is mixed with \( 1 \frac{1}{2} \) quarts of water to make lemonade for 6 people. If you use the same proportions of concentrate and water, how many quarts of lemonade concentrate are needed to make lemonade for 21 people?

15. If 25 percent of 300 is equal to 500 percent of \( t \), then \( t \) is equal to what number?

16. In the figure below, what is the area of the shaded square in square units?

![Shaded square]

17. There are 140 students in the school band. One of these students will be selected at random to be the student representative. If the probability that a brass player is selected is \( \frac{2}{5} \), how many brass players are in the band?

18. A shelf holds fewer than 50 cans. If all of the cans on this shelf were put into stacks of five cans each, no cans would remain. If the same cans were put into stacks of three cans each, one can would remain. What is the greatest number of cans that could be on the shelf?

Part 3  Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- the quantity in Column A is greater;
- the quantity in Column B is greater;
- the two quantities are equal;
- the relationship cannot be determined from the information given.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{4} )</td>
<td>( \frac{4}{3} )</td>
</tr>
<tr>
<td>( \frac{3^2}{4} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 3s )</td>
</tr>
<tr>
<td>( 120 )</td>
<td>( 2a )</td>
</tr>
</tbody>
</table>

23. The average (arithmetic mean) of \( s \) and \( t \) is greater than the average of \( s \) and \( w \).